Integrating the Planck Fctn.

$$B(T) = \frac{2h}{c^2}\int_0^\infty \frac{\nu^3\,d\nu}{e^{h\nu/kT}-1}$$

Let $x = h\nu/kT \Rightarrow \nu = \frac{kT}{h}\cdot x \Rightarrow d\nu = \frac{kT}{h}dx$

$$B(T) = \frac{2h}{c^2}\cdot\left(\frac{kT}{h}\right)^4\int_0^\infty \frac{x^3\,dx}{e^x-1} \longleftarrow$$ Let's integrate this function

$$\int_0^\infty \frac{x^3\,dx}{e^x-1}\times\frac{e^{-x}}{e^{-x}} = \int_0^\infty \frac{x^3 e^{-x}}{1-e^{-x}}dx = \int_0^\infty x^3 e^{-x}\cdot\frac{1}{1-e^{-x}}dx$$

Remember that $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$

so $\frac{1}{1-e^{-x}} = 1 + e^{-x} + e^{-2x} + e^{-3x} + \cdots$

So our integral becomes $\int_0^\infty x^3 e^{-x}\cdot\sum_{n=0}^\infty e^{-nx}\,dx = \int_0^\infty x^3\cdot\sum_{n=1}^\infty e^{-nx}\,dx$

$x\,e^{-x}$ into sum

Let's look @ each term in sum (we can do this because each term in the sum is a convergent integral).

Let $u = nx \Rightarrow x = \frac{u}{n} \Rightarrow dx = \frac{du}{n}$

$$\int_0^\infty x^3 e^{-nx}\,dx = \frac{1}{n^4}\int_0^\infty u^3 e^{-u}\,du = \frac{\Gamma(4)}{n^4} = \frac{6}{n^4}$$

$\left\{\begin{array}{l}\text{NOTE: GAMMA FUNCTION} \\ \Gamma(z) \equiv \int_0^\infty x^{z-1}e^{-x}\,dx \\ \Gamma(n) = (n-1)!\end{array}\right.$

Now we put back the sum so the total integral equals:

$$\int_0^\infty \frac{x^3}{e^x-1}dx = \sum_{n=1}^\infty \frac{6}{n^4} = 6\cdot\zeta(4) = 6\cdot\frac{\pi^4}{90}$$

$\left\{\begin{array}{l}\text{NOTE: Riemann Zeta Function} \\ \zeta(s) = \sum_{n=1}^\infty \frac{1}{n^s}\end{array}\right.$

$$B(T) = \frac{2\pi^4 k^4}{15 c^2 h^3}T^4$$ $\mathrm{erg\cdot s^{-1}\cdot cm^{-2}\cdot ster^{-1}}$