6. The brightness temperature, $T_B$, of a source is the temperature that is proportional to the intensity given by the Planck function, $B_V(T_B)$, in the Rayleigh-Jeans limit ($h\nu/kT_B \ll 1$). Radio telescopes don’t actually measure specific intensity, but instead measure the flux density of the source observed within the diffraction beam, called the Power Pattern, of the telescope. This Power Pattern (also “beam pattern”) is the sensitivity of the telescope to radiation coming from different directions. The main diffraction lobe of a typical radio telescope has a Power Pattern, $P_n(\theta,\phi)$, that is characterized by a Gaussian function with the Full Width Half Max (FWHM) beamwidth of $\theta_{\text{fwhm}}$ (see Figure below) and $P_n(\theta,\phi) = \exp(-4 \ln(2) \theta^2 / \theta_{\text{fwhm}}^2)$. Derive the equation for how $T_B$ is related to observed flux density $F_V$ assuming $T_B$ is constant across the telescope beam solid angle (write your answer in terms of $\theta_{\text{fwhm}}$). This expression is handy for going from K to Janskys (Jansky is the unit of flux density used in radio astronomy: $1 \text{ Jy} = 10^{-23} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1}$). Hints: Radio telescope $\theta_{\text{fwhm}}$ are usually small angles ($\theta_{\text{fwhm}} \sim 1.2\lambda/D \sim 67''$ for the Kitt Peak 12m at 90 GHz). The solid angle of the telescope beam is not simply $\Omega = \pi \theta_{\text{fwhm}}^2 / 4$. Got to integrate...

![Figure 1: A radio telescope diffraction pattern. The angle that corresponds to where the sensitivity drops from 1 to 0.5 in the main lobe is called the FWHM (labeled Half Power Beamwidth). Radio telescope main lobes (labeled “A”) are usually well approximated by a Gaussian function of angle $\theta$. They are normalized such that $P_n(\theta,\phi) = 1$ in the direction the telescope is pointing.](image)
7. Protostars are classified (Class 0/I/II/III) by how deeply embedded they are within the dusty cores in which they form within molecular clouds. One evolutionary metric is the “Bolometric Temperature” \( (T_{\text{bol}}) \) which is defined as the temperature of the blackbody that has the same mean frequency as the observed Spectral Energy Distribution (SED) of the protostar. In a seminal paper by Chen et al. 1995 ApJ 445, 377, \( T_{\text{bol}} \) was determined for protostars in different evolutionary phases. Derive their equation:

\[
T_{\text{bol}} = \frac{\zeta(4)}{4\zeta(5)} \frac{h\langle v \rangle}{k} = 1.25 \times 10^{-11} \langle v \rangle \text{ K Hz}^{-1}
\]

Hint: To calculate the mean (average) frequency of a Planck function, think about the continuous analogy to weighted averaging.

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Bolometric luminosity-temperature diagrams of all 235 YSOs in the three regions. The open circles are the sources with a known spectral type, and the stars are the sources without a known spectral type. The solid line shows the zero-age main sequence for stellar mass of 0.4–6 \( M_\odot \). Sources without peaks in the infrared are shown with the arrows. The dashed lines show the regimes approximately corresponding to the YSO spectral energy distribution classes 0, I, II,