AST 300B – Spring 2019

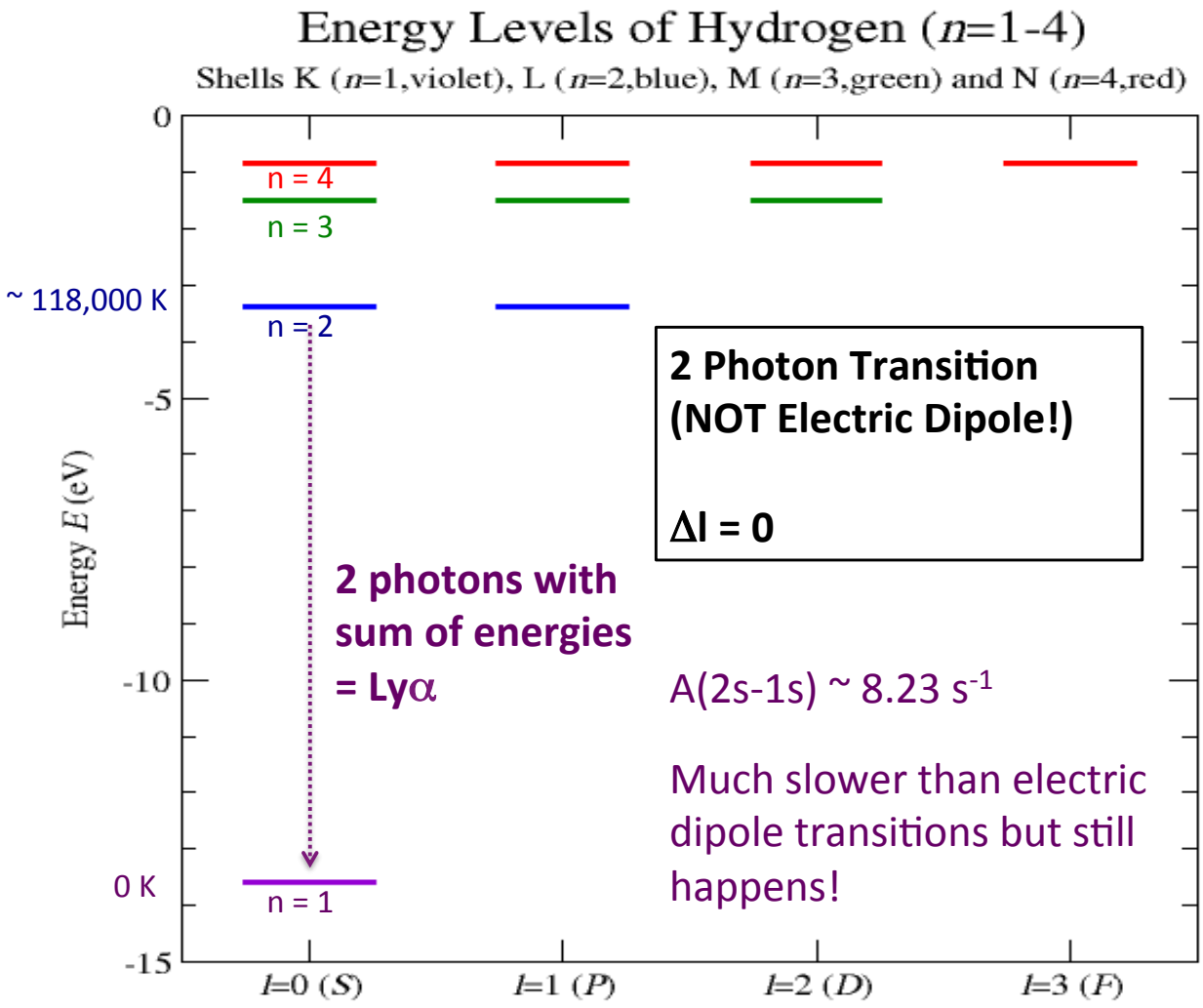
In-class/take-home Problems Due: Wed. April 3rd

31. The emissivity coefficient j_ν (erg s^{-1} cm^{-3} $ster^{-1}$ Hz^{-1}) for the two-photon transition 2s – 1s in the Hydrogen atom has the same equation as for an emission line except that the line profile function is replaced by $P_{2s}(\nu)$ which is the probability that a photon is emitted at frequency ν. The Einstein A(2s-1s) ~ 8.23 s^{-1}. A Hydrogen atom energy level diagram is on the back.

a. In the low density limit, collisional excitation and de-excitation are negligible, and the only way to populate the 2s level is by recombination at an effective recombination rate denoted by α_{2s}^{eff}. Write down the expression for j_ν in terms of $P_{2s}(\nu)$ and α_{2s}^{eff} assuming statistical equilibrium ($dn_{2s}/dt = 0$). [Hints: You will need to use statistical equilibrium to relate n_{2s} to α_{2s}^{eff}. What can you do with the Einstein B terms in the high ν limit?]

b. It is possible that the density in HII regions can become high enough that the 2s level is de-excited by collisions with protons and electrons to the 2p level. The "downward" collision rates are given by $\gamma(2s\text{-}2p) = 4.74 \times 10^{-4}$ cm^3 s^{-1} for collisions with protons and by $\gamma(2s\text{-}2p) = 0.57 \times 10^{-4}$ cm^3 s^{-1} for collisions with electrons (Osterbrock 1974). What is the critical density (n_{crit} cm^{-3}) for the 2s-1s transition?

c. Given that the critical density in part (b) is not unreasonable for HII regions, we can't always assume that the low density limit applies. Modify your equation for j_ν assuming that collisional de-excitation by protons and electrons from 2s-2p cannot be ignored and assuming statistical equilibrium ($dn_{2s}/dt = 0$). [Hint: You don't need to worry about collisions from 2p-2s (because there is a ***very fast*** Einstein A from 2p-1s) or between 2s and any other levels. Also, what can you assume about the number density of electrons and protons?]. Express your answer in terms of n_{crit}.


Energy Levels of Hydrogen (n=1-4)
Shells K (n=1,violet), L (n=2,blue), M (n=3,green) and N (n=4,red)
0
-5
-10
-15
Energy E (eV)
n = 4
n = 3
~ 118,000 K
n = 2
0 K
n = 1
l=0 (S)
l=1 (P)
l=2 (D)
l=3 (F)
2 Photon Transition
(NOT Electric Dipole!)
Δl = 0
2 photons with
sum of energies
= Lyα
A(2s-1s) ~ 8.23 s-1
Much slower than electric
dipole transitions but still
happens!