AST 300B – Spring 2019
<u>In-class/homework Problems Due: Friday Mar 29</u>

30. In a previous homework problem, you proved that singly-ionized carbon (CII or C^+) may be abundant in neutral atomic H environments because the ionization potential of C is 11.4 eV, less than the 13.6 eV ionization potential of H. One of the strongest spectral lines in star-forming galaxies is the ~158 μm (more accurately ν = 1900.5369 GHz) far-infrared transition of CII. CII has 2 fine structure levels in the ground electronic state which can be well approximated as a 2 level system (all CII atoms in one of those two energy levels). The statistical weights of the upper and lower levels are $g_u = 4$ and $g_l = 2$ and the energy levels are $(E/k)_u = 91.21$ K and $(E/k)_l = 0$ K. $A_{ul} = 2.36 \times 10^{-6}\ s^{-1}$. The study of Barinovs et al. (2005) finds $\gamma_{ul}(T) \sim 3.8 \times 10^{-10}\ T^{0.15}\ cm^3\ s^{-1}$.

(a) The "integrated intensity", I, is defined as the integral over frequency of the monochromatic specific intensity of the spectral line (units of I = erg s^{-1} cm^{-2} $ster^{-1}$; notice the Hz^{-1} has been integrated out). Solve the radiative transfer equation ***in the optically thin limit*** and derive how the integrated intensity, I, depends on the frequency of the transition, the Einstein A, and the column density in the upper state of the transition, and some constants. Assume that the level populations of the CII atoms are constant along the line of sight. NOTE: you have two integrals in this problem – an integral over frequency and an integral over line of sight. Both integrals are easy - don't make them hard.

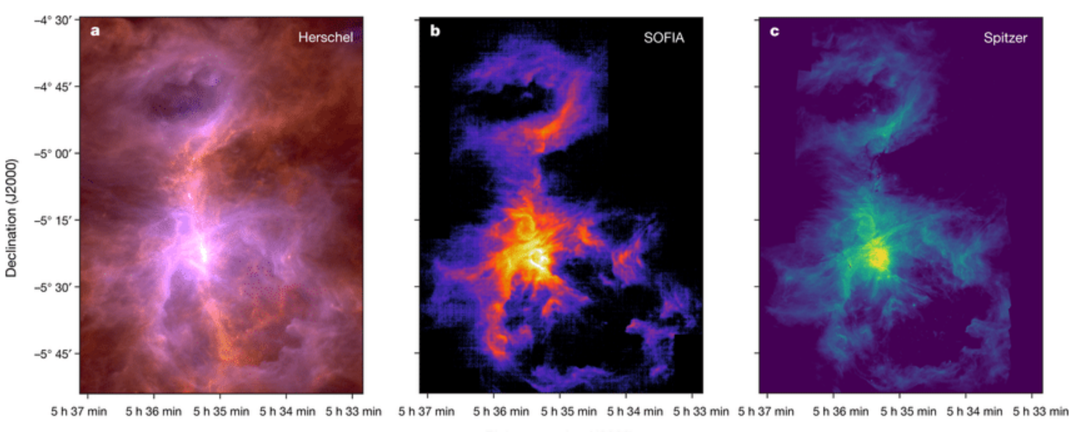

Figure 1: Three views of the region around the Orion Nebula: (a) far-infrared/submillimeter thermal dust emission. (b) CII 158 μm emission, (c) PAH emission at 8 μm.

(b) Assume you have a HI cloud with a density of $n_H = 10^3$ cm^{-3}, a gas kinetic temperature of $T_K = 100$ K, and assume the CMB is the dominant radiation background. Calculate the excitation temperature of the 158 μm transition of CII in this cloud assuming statistical equilibrium. [Hint: what can you assume about the stimulated emission rates in this problem? Look back at a previous homework.]

(c) Calculate the critical density of the CII 158 μm line. How does the density of this HI cloud compare with the critical density (quote your answer as a ratio)?

(d) If you observe the integrated intensity of the CII 158 μm line in this cloud to be I = 1 x 10^{-4} erg s^{-1} cm^{-2} ster^{-1}, then calculate the total column density of CII (cm^{-2}). NOTE: this is not just the column density in the upper energy level, but the total column density for both energy levels ($N(CII) = N_l + N_u$). Boltzmann is your buddy.

(e) Up to this point, we've assumed that the emission was optically thin. Let's check that assumption by setting $\tau_\nu = 1$. Calculate the column density for which the optical depth at the peak of the spectral line (v = 0 km/s) is equal to 1 for the CII 158 μm line with a velocity dispersion $\sigma_v = 3$ km/s and constant excitation temperature along line of sight (using T_{ex} you calculated above in part-b). Was optically thin an ok assumption? Assume the line profile function is given by a Gaussian function of velocity v (which is what you would expect for Doppler motions in a gas). Note: this equation has both velocity v ***and*** frequency ν. Frequency is only in the term c/ν):

$$\phi_\nu = (1/\mathrm{sqrt}(2\pi)) * (c/\nu) * (1/\sigma_v) * \exp(-v^2/2\sigma_v^2)$$

Hints: Use the definition of τ_ν for a spectral line and convert all Einstein Bs to As. You should have 2 terms - pull out a factor of N_u, the column density in the upper state, and use the Boltzmann equation to simplify what's left over.