17. Let’s calculate the long wavelength “spectral index” of dust emission. The spectral index, \( \alpha \), is defined as the power-law slope of the observed flux density over some range of wavelengths: \( F_\nu \sim \nu^\alpha \). Consider an optically thin source of dust emission with emission well described by a source function \( B_\nu(T_d) \) over a solid angle \( \Omega \), and dust mass opacity parametrized by \( \kappa(\nu) = \kappa_0 \left( \nu / \nu_0 \right)^\beta \) (cm\(^2\)/g of dust) at long wavelengths. Assume that the source is small and \( F_\nu = I_\nu \Omega \) is a good approximation.

(a) How does the dust optical depth \( \tau_\nu \) depend on frequency?

(b) In the Rayleigh-Jeans limit at long wavelength what is the spectral index \( \alpha \) for (1) blackbody emission and (2) for optically thin dust emission with a dust opacity index of \( \beta \)?