16. Consider an optically thin dusty disk surrounding a young protostar with luminosity $L_{\text{star}}$. In this problem, you will derive how the temperature of dust grains varies with distance, $r$, from the star.

(a) Assume the dust opacity at long wavelengths is given by a power-law $Q_{\text{abs}} \sim \nu^\beta$. Derive the dependence of $T_d$ with $r$ and $\beta$. I just want you to write down how $T_d$ is proportional to $r$ and $\beta$ (don't have to write out all the constants, etc.).

(b) In typical ISM dust $\beta \sim 2$, but in young protostellar disks where dust grain start to grow into mm sized grains ("dust coagulation"), we find that $\beta \sim 1$. Large (km) planetesimals would radiate closer to a blackbody and would have $\beta \sim 0$. Thus, there is significant evolution in the opacity ($Q_{\text{abs}}$ or $\kappa_\nu$) of dust grains in planet-forming disks (see Figure below for a calculation of how $\beta$ at long wavelengths varies with size of grains). Calculate the power-law indices of $T_d$ with $r$ for $\beta = 2, 1, \text{and} 0$ (simplify any fractions).