Einstein Coefficients

Let's analyze 2 level system in limit of $n_c \to 0$

$$n_1 B_{12} u_\nu = n_2 A_{21} + n_2 B_{21} u_\nu$$

Solve for u_ν and divide both sides by $n_2 B_{21}$:

$$\frac{n_1 B_{12}}{n_2 B_{21}} u_\nu - u_\nu = \frac{A_{21}}{B_{21}}$$

$$u_\nu = \frac{A_{21}}{B_{21}} \cdot \frac{1}{\frac{B_{12}}{B_{21}}\frac{n_1}{n_2} - 1}$$

Boltzmann: $\frac{n_1}{n_2} = \frac{g_1}{g_2} e^{+h\nu/kT}$ because $\Delta E_{21} = h\nu$

$$u_\nu = \frac{A_{21}}{B_{21}} \cdot \frac{1}{\frac{B_{12}}{B_{21}}\frac{g_1}{g_2} e^{h\nu/kT} - 1}$$

But $u_\nu = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/kT} - 1}$ for a black body

$\Rightarrow$ $A_{21} = \frac{8\pi h\nu^3}{c^3} B_{21}$ and $g_1 B_{12} = g_2 B_{21}$

CAN CONVERT Einstein Bs to As and vice versa!

Note we derived this in a particular limit assuming u_ν was a blackbody — <u>BUT</u> these are <u>consts</u> so are always true.

SEMI-CLASSICAL Derivation of Einstein A ASTR 300B

The exact equation depends on the "multipole" of radiation.
Radiation is generated by acceleration of charge and the "Strongest" multipole is Electric Dipole Radiation.

From Quantum point-of-view $\langle P \rangle = h\nu \cdot A_{ul}$

$\text{erg} \cdot \text{s}^{-1}$ $\quad$ $\text{erg} \cdot \text{s}^{-1}$ ✓

Lets relate this average power radiated to the average power from as oscillating electric dipole. $d = e \cdot x(t)$

electric dipole = charge · distance

$x(t) = x_0 \cos \omega t$ where $\omega = 2\pi\nu$

$\Rightarrow \ddot{x}(t) = x_0 \omega^2 \cos \omega t$ is acceleration of the charge

The Power radiated into 4π ster from accelerating charge is:

$$P(t) = \frac{2}{3} \frac{e^2 \ddot{x}(t)}{c^3}$$

$\int_0^{2\pi/\omega} \cos^2 \omega t \, dt = \frac{\pi}{\omega}$ ← one period of oscillation

$$\langle P \rangle = \frac{2}{3c^3} \cdot e^2 x_0^2 (2\pi\nu)^4 \cdot \frac{\int_0^{2\pi/\omega} \cos^2 \omega t \, dt}{\int_0^{2\pi/\omega} dt} \qquad \int_0^{2\pi/\omega} dt = \frac{2\pi}{\omega}$$

$$\langle P \rangle = \frac{2 \cdot 16 \cdot \pi^4 \nu^4}{3c^3} \cdot \frac{\pi/\omega}{2\pi/\omega} e^2 x_0^2$$

we almost have that. need to $\times \frac{2}{2}$

Define the average electric dipole as $\bar{\mu}_e = \frac{e x_0}{2} \Rightarrow \mu_e^2 = \frac{e^2 x_0^2}{4}$

$$\langle P \rangle = \frac{64 \pi^4 \cancel{\nu^4}\nu^3}{3c^3} \bar{\mu}_e^2 = h\cancel{\nu} A_{ue}$$

$$\Rightarrow A_{ue} = \frac{64\pi^4 \nu^3}{3hc^3} |\mu_{ue}|^2$$

s^{-1}

electric dipole moment matrix element $= \iiint \psi_i^* \hat{\mu}_e \psi_f \, dV$

units $[\mu_e] \Rightarrow 1$ Debye $= 10^{-18}$ esu·cm (cgs units)

NOTE Electric Dipole $\sim \nu^3$ (also magnetic dipole)

TURNS OUT THAT Electric Quadrupole $\sim \nu^5$ (also magnetic quadrupole) etc.