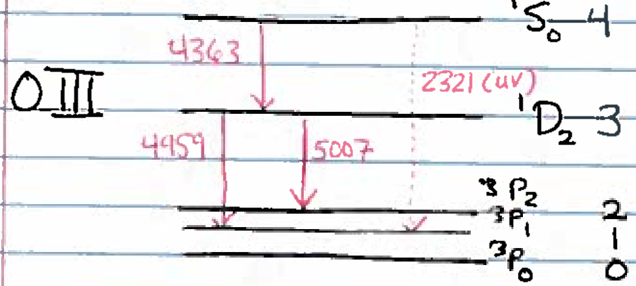
HII Region Probes - Forbidden Lines

ASTR 300B

T-sensitive line ratios np^2 atoms



3 optical forbidden lines (1 in uv difficult to observe)

Assume ① low n such that every collisions to S or D level radiatively de-excites

② high ν limit (ignore B terms)

③ Optically thin $j_\nu = \frac{h\nu}{4\pi} n_u A_{ul}$ $I_\nu = \int_{los} j_\nu ds$

$$\frac{j(4959) + j(5007)}{j(4363)} = \frac{j_{32} + j_{31}}{j_{43}} = \frac{h\nu_{32} A_{32} n_D + h\nu_{31} A_{31} n_D}{h\nu_{43} A_{43} n_S}$$

Let's use statistical equilibrium to determine this ratio

$$= \frac{n_D}{n_S} \frac{A_{31}\nu_{31} + A_{32}\nu_{32}}{A_{43}\nu_{43}}$$

④ Statistical equilibrium with assumption #1 (implies $A_{43} n_S \ll n_e n_p \gamma_{PD}$)

Note: Don't worry about recombination because OIV not present in HII regions

$$\frac{dn_D}{dt} = 0 \Rightarrow n_e n_p \gamma_{PD} = n_D (A_{31} + A_{32})$$

$n_p = n_{^3P_0} + n_{^3P_1} + n_{^3P_2}$ collision rate from all ^3P_J levels to ^1D

$$\frac{dn_S}{dt} = 0 \Rightarrow n_e n_p \gamma_{PS} = n_S (A_{43} + A_{41})$$

Taking the ratio of these equations gives

$$\frac{n_D}{n_S} = \frac{n_D/n_p}{n_S/n_p} = \frac{\gamma_{PD}}{\gamma_{PS}} \frac{A_{43} + A_{41}}{A_{31} + A_{32}}$$

Remember: collision rates with e^- can be written as:

$$\gamma_{ul} \simeq 8.63 \times 10^{-6}\ cm^3 \cdot s^{-1} \frac{\Omega(u,l)}{g_u T_K^{1/2}}$$ ← "e^- collision strength"

(look back in notes...)

USING the fact that $\gamma_{\ell u} = \gamma_{u\ell} \frac{g_u}{g_\ell} e^{-\Delta E_{u\ell}/k T_K}$

then

$$\frac{\gamma_{PD}}{\gamma_{PS}} = \frac{\Omega(P,D)}{\Omega(P,S)} \cdot \frac{\frac{\cancel{g_D}}{\cancel{g_P}} \cdot \frac{1}{\cancel{g_D}} \, e^{-\Delta E_{DP}/k T_K}}{\frac{\cancel{g_S}}{\cancel{g_P}} \cdot \frac{1}{\cancel{g_S}} \, e^{-\Delta E_{SP}/k T_K}} = \frac{\Omega(P,D)}{\Omega(P,S)} e^{+\Delta E_{SD}/k T_K}$$

all statistical weights cancel!

Substituting into emissivity ratio equation:

$$\frac{j_{32} + j_{31}}{j_{43}} = \frac{\Omega(P,D)}{\Omega(P,S)} \cdot \frac{A_{43} + A_{41}}{A_{31} + A_{32}} \; \frac{A_{31}\nu_{31} + A_{32}\nu_{32}}{A_{43}\nu_{43}} \; e^{\Delta E_{SD}/k T_K}$$

$\Rightarrow$ Depends on $e^{\Delta E_{SD}/k T_K} \sim e^{32000/k T_K}$ $\Rightarrow$ good T_K probe of $T_K \sim 10^4$ K gas!

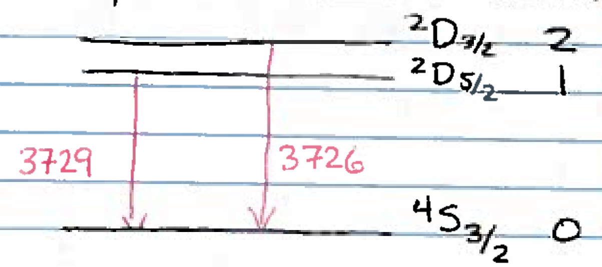
NOTE our Assumption #1 is valid for $n < n_{crit}$

$n_{crit} \sim 10^{4-5}$ cm^{-3} for typical np^2 forbidden lines

For collisions with e^- $\quad \frac{\Omega(P,D)}{\Omega(P,S)} = \frac{2.50}{0.3}$

$$\frac{I(4959) + I(5007)}{I(4363)} \simeq 8.32 \, e^{\Delta E_{SD}/k T_K}$$

Density-sensitive line ratios $\quad np^3$ atoms



} Closely spaced fine structure splitting

$n_{crit}(3729) \sim 3.3\times10^3\ cm^{-3}$ @ 10^4 K
$n_{crit}(3726) \sim 4.5\times10^3\ cm^{-3}$

First let's explore the low n limit $\Rightarrow$ only Radiative decays important.

$$\frac{j(3729)}{j(3726)} = \frac{j_{10}}{j_{20}} = \frac{\cancel{h\nu_{10}}\, n_1 A_{10}}{\cancel{h\nu_{20}}\, n_2 A_{20}} \quad (\simeq 1)$$

$\frac{\nu_{10}}{\nu_{20}} \simeq 1$ because closely spaced fine structure splitting!

Statistical equilibrium:

$$\frac{dn_2}{dt} = 0 \Rightarrow n_e n_S \gamma_{^4S,^2D_{3/2}} = n_2 A_{20}$$

$$\frac{dn_1}{dt} = 0 \Rightarrow n_e n_S \gamma_{^4S,^2D_{5/2}} = n_1 A_{10}$$

$$\frac{n_1}{n_2} = \frac{\gamma(^4S,{}^2D_{5/2})}{\gamma(^4S,{}^2D_{3/2})}\frac{A_{20}}{A_{10}} = \frac{A_{20}}{A_{10}} \cdot \frac{\Omega(^4S,{}^2D_{5/2})\,\dfrac{\cancel{g(^2D_{5/2})}}{g(^4S_{3/2})}\dfrac{1}{\cancel{g(^2D_{5/2})}}}{\Omega(^4S,{}^2D_{3/2})\,\dfrac{\cancel{g(^2D_{3/2})}}{g(^4S_{3/2})}\dfrac{1}{\cancel{g(^2D_{3/2})}}}$$

again, all stat. weights cancel!

$$\times \cancel{\exp\left(\frac{\Delta E(^2D_{5/2}-{}^2D_{3/2})}{k T_K}\right)} \quad (\simeq 1)$$

Notice that because fine structure lines so close in ΔE, T_K dependence DROPS OUT!

Now we need a property of Ω which I quote here without proof:

$$\Omega(SLJ, S'L'J) = \Omega(SL, S'L')\frac{(2J'+1)}{(2S'+1)(2L'+1)}$$

$\uparrow$ collision strength summed over all J

$$\frac{\Omega(^4S,{}^2D_{5/2})}{\Omega(^4S,{}^2D_{3/2})} = \frac{2\cdot\frac{5}{2}+1}{2\cdot\frac{3}{2}+1} = \frac{6}{4} = \frac{3}{2}$$

Substituting for collision rates:

$$\frac{I(3729)}{I(3726)} = \frac{3}{2} \cdot \frac{\cancel{A_{20}}}{\cancel{A_{10}}} \cdot \frac{\cancel{A_{10}}}{\cancel{A_{20}}} \rightarrow \frac{3}{2} \text{ in low } n \text{ limit!}$$

What about the high n limit?
Then collisional equilibrium sets level populations

$$\frac{n_1}{n_2} = \frac{g_1}{g_2} \cdot e^{\cancel{\frac{-\Delta E(^2D_{5/2} - ^2D_{3/2})}{kT_K}}} \simeq 1$$

$$g_1 = g(^2D_{5/2}) = 2J+1 = 2 \cdot \tfrac{5}{2} + 1 = 6$$

$$g_2 = g(^2D_{3/2}) = 2J+1 = 2 \cdot \tfrac{3}{2} + 1 = 4$$

$$\frac{I(3729)}{I(3726)} = \frac{6}{4} \cdot \frac{A_{10}}{A_{20}} = \frac{3}{2} \frac{3.6 \times 10^{-5}}{1.6 \times 10^{-4}} \simeq 0.34 \text{ low } n \text{ limit}$$

So this line ratio varies from 1.5 to 0.34 and is sensitive to density in between these limits!

To solve for exact curve, you have to do the statistical equilibrium calculation with collisional de-excitation terms from $^2D_{5/2}$, $^2D_{3/2}$ included.

General Rule of thumb: transitions from energy levels separated by $\Delta E \sim$ few $\times kT_K$ are good T_K probes while transitions from energy levels with $\Delta E \ll kT_K$ are good n probes. All probes only work over a limited range in density.