H II Region Probes - Forbidden Lines

\( T \)-sensitive line ratios

\[ \begin{align*}
    & \text{O III} \\
    & \text{[3p]}_2 \rightarrow [3p]_1 \\
    & \text{[3p]}_1 \rightarrow [3s]_2 \\
    & \text{[3s]}_2 \rightarrow [3s]_1 \\
\end{align*} \]

np\(^2\) atoms

3 optical forbidden lines
(1) In UV difficult to observe
(2) High \( \nu \) limit (ignore B term)
(3) Optical thin

\[ j_x = \frac{h \gamma_x}{\pi n} \text{nu} A_{x2} \]

\[ I_x = \int_{10^5} j_x ds \]

\[ \frac{j(4363) + j(5007)}{j(4959)} = \frac{j_{32} + j_{31}}{j_{43}} = \frac{h \gamma_{32} A_{32} n_D + h \gamma_{31} A_{31} n_D}{h \gamma_{43} A_{43} n_S} \]

Let's use statistical equilibrium to determine this ratio:

\[ \frac{n_D}{n_S} = \frac{A_{31} \gamma_{31} + A_{32} \gamma_{32}}{A_{43} \gamma_{43}} \]

(4) Statistical equilibrium with assumption #1 (implies \( A_{43} n_S \ll n_e n_p \sigma_{PD} \))

\[ \frac{dn_D}{dt} = 0 \Rightarrow n_e N_p \sigma_{PD} = n_D (A_{31} + A_{32}) \]

\[ n_p = n_{3p} + n_{3p} + n_{3p} \]

\[ \frac{dn_s}{dt} = 0 \Rightarrow n_e n_p \sigma_{PS} = n_S (A_{43} + A_{44}) \]

Taking the ratio of these equations gives:

\[ \frac{n_D}{n_S} = \frac{n_D/n_p}{n_S/n_p} = \frac{\sigma_{PD}}{\sigma_{PS}} \frac{A_{43} + A_{44}}{A_{31} + A_{32}} \]

Remember: collision rates with e\(^-\) can be written as:

\[ \sigma_{ul} \sim 8.63 \times 10^{-6} \text{ cm}^2 \text{ s}^{-1} \]

\[ \frac{\sigma(\text{e}^-)}{n_e T^{1/2}} \]

(look back in notes...)

Note 2:
Don't worry about recombination because O III not present in H II regions.
Using the fact that \( \Delta e_{eu} \approx \Delta e \cdot \frac{g_u}{g_v} \cdot e^{-\frac{\Delta E_{eu}}{k_B T_k}} \)

then

\[
\frac{\Delta p}{\Delta p_s} = \frac{\Omega(P,D)}{\Omega(P,S)} \cdot \frac{\frac{g_P}{g_S}}{\frac{g_D}{g_S}} \cdot e^{-\frac{\Delta E_{DP}}{k_B T_k}} = \frac{\Omega(P,D) + \Delta E_{DP}}{\Omega(P,S) \cdot e^{\frac{\Delta E_{DP}}{k_B T_k}}}
\]

Note that all statistical weights cancel.

Substituting into emissivity ratio equation

\[
\frac{J_{32} + J_{31}}{J_{43}} = \frac{\Omega(P,D)}{\Omega(P,S)} \cdot \frac{A_{43} + A_{41}}{A_{31} + A_{32}} \cdot \frac{A_{31} \gamma_{31} + A_{32} \gamma_{32}}{A_{43} \gamma_{43}} \cdot e^{\frac{\Delta E_{SD}}{k_B T_k}}
\]

\[\Rightarrow \text{Depends on } e^{\frac{\Delta E_{SD}}{k_B T_k}} \sim e^{\frac{32000}{k_B T_k}} \Rightarrow \text{good } T_k \text{ probe of } T_k \sim 10^3 \text{ K gas.}
\]

\text{Note our Assumption #1 is valid for } N < N_{\text{crit}} \quad N_{\text{crit}} \sim 10^{4.5} \text{ cm}^{-3} \text{ for typical np z forbidden lines}

For collisions with \( e^{-\frac{\Omega(P,D)}{\Omega(P,S)} = \frac{2.50}{0.3}} \)

\[
\frac{I(4959) + I(5007)}{I(4363)} \sim 8.32 e^{\frac{\Delta E_{SD}}{k_B T_k}}
\]
Density-sensitive line ratios $n \rho^3 \text{ atoms}$

$^2D_{3/2} - ^2D_{5/2}$

$3^2\text{ Structure splitting}$

$3729 \quad 3726$

$n_{e \text{ init}}(3729) \approx 3.3 \times 10^3 \text{ cm}^{-3}$

$n_{e \text{ init}}(3726) \approx 4.5 \times 10^3 \text{ cm}^{-3}$

First, let's explore the low $n$ limit $\Rightarrow$ only radiative decays important.

$$\frac{j_{3729}}{j_{3726}} = \frac{j_{10}}{j_{20}} \Rightarrow K_{20} A_{10} = \frac{K_{10} n_1 A_{10}}{K_{20} n_2 A_{20}}$$

Statistical equilibrium gives

$$\frac{dn_1}{dt} = 0 \Rightarrow n_e n_5 \langle \sigma v \rangle_{4S, ^2D_{3/2}} = n_2 A_{20}$$

$$\frac{dn_1}{dt} = 0 \Rightarrow n_e n_5 \langle \sigma v \rangle_{4S, ^2D_{5/2}} = n_1 A_{10}$$

$$\frac{n_1}{n_2} = \frac{\langle \sigma v \rangle_{4S, ^2D_{3/2}}}{\langle \sigma v \rangle_{4S, ^2D_{5/2}}} \frac{A_{20}}{A_{10}} = \frac{A_{20}}{A_{10}} \frac{\langle \sigma v \rangle_{4S, ^2D_{3/2}}}{\langle \sigma v \rangle_{4S, ^2D_{5/2}}} \frac{1}{g(4S, ^2D_{3/2}) g(4S, ^2D_{5/2})} \times \exp \left( \frac{\Delta E(4S, ^2D_{3/2})}{kT} \right)$$

Notice that because fine structure lines so close in $\Delta E$, $T_k$ dependence drops out.

Now we need a property of $\Sigma$ which I quote here without proof:

$$\Sigma (SLLJ, S'L', J') = \Sigma (SLLJ, S'L') \frac{(2J' + 1)}{(2S' + 1)(2L' + 1)}$$

$$\Sigma (4S, ^2D_{3/2}) = \frac{2 \cdot \frac{3}{2} + 1}{2 \cdot \frac{3}{2} + 1} = \frac{6}{4} = \frac{3}{2}$$

$$\Sigma (4S, ^2D_{5/2}) = \frac{2 \cdot \frac{5}{2} + 1}{2 \cdot \frac{3}{2} + 1} = \frac{6}{4} = \frac{3}{2}$$
Substituting for collision rates:

\[
\frac{I(3729)}{I(3726)} = \frac{3}{2} \cdot \frac{A_{10}}{A_{20}} \rightarrow \frac{3}{2} \text{ in low } n \text{ limit}
\]

What about the high \( n \) limit?
Then collisional equilibrium sets level populations

\[
\frac{n_1}{n_2} = \frac{g_1}{g_2} \cdot e^{-\frac{\Delta E}{k_T}} \approx 1
\]

\( g_1 = g\left( ^2D_{5/2} \right) = 2J + 1 = 2 \cdot \frac{5}{2} + 1 = 6 \)

\( g_2 = g\left( ^2D_{3/2} \right) = 2J + 1 = 2 \cdot \frac{3}{2} + 1 = 4 \)

\[
\frac{I(3729)}{I(3726)} = \frac{6}{4} \cdot \frac{A_{10}}{A_{20}} = \frac{3}{2} \cdot \frac{3.6 \times 10^{-5}}{1.6 \times 10^{-4}} \approx 0.34 \text{ low } n \text{ limit}
\]

So this line ratio varies from 1.5 to 0.34 and is sensitive to density in between these limits.

To solve for exact curve, you have to do the statistical equilibrium calculation with collisional de-excitation terms from \(^2D_{5/2}, ^2D_{3/2}\) included.

**General Rule of thumb:** transitions from energy levels separated by \( \Delta E \approx k_B T_k \) are good \( T_k \) probes while transitions from energy levels with \( \Delta E \ll k_B T_k \) are good \( n \) probes. All probes only work over a limited range in density.