Stellar Energy Sources

ASTR 250

Over the ~ 4.6 Billion year lifetime of the Sun it has radiated:

$$E_\odot^{life} \approx L_\odot \cdot t_{life}$$

$$\sim 3.9 \times 10^{33} \text{ erg} \cdot s^{-1} \cdot 4.6 \times 10^{9} \text{ yrs} \cdot \frac{3.1 \times 10^7 s}{\text{yrs}}$$

$$\sim 5.5 \times 10^{50} \text{ erg}$$

What supplies this energy?

Gravitational Contraction?

$$U_{grav} = -\frac{3}{5}\frac{GM^2}{R}$$ for uniform density sphere

Virial theorem state $\frac{1}{2} U_{grav}$ radiated away as Sun collapsed to present radius:

$$E_\odot^{grav} = -\frac{3}{10}\frac{GM^2}{R} \sim 1.2 \times 10^{48} \text{ erg}$$

$$t_{KH} = \frac{E_\odot^{grav}}{L_\odot} = \frac{1.2 \times 10^{48} \text{ erg}}{3.9 \times 10^{33} \text{ erg} \cdot s^{-1}} = 3.1 \times 10^{14} s \cdot \left(\frac{1 \text{ yr}}{3.1 \times 10^7 s}\right)$$

$$\sim 10^7 \text{ years}$$

This timescale is called the "Kelvin-Helmholtz" timescale.

$$t_{KH} << t_{life}.$$

$$E_\odot^{grav} << E_\odot^{life}$$

Chemical Reactions? "Burning/Combustion"

Assume the Sun consists of H and O in the optimum proportions to be burned to form H_2O vapor

H_2O molecular weight $= m(^{16}O) + 2m(^1H)$

$$\sim 18 \cdot m_H$$
$$\sim 18 \cdot (1.67 \times 10^{-24}\ g)$$
$$\sim 3.5 \times 10^{-23}\ g$$

If the entire Sun combusted then

$$N_{H_2O} = \frac{M_\odot}{\text{weight of } H_2O \text{ molecule}} = \frac{2 \times 10^{33}\ g}{3.5 \times 10^{-23}\ g} \sim 6 \times 10^{55}\ H_2O \text{ molecules}$$

Combustion releases about 1-2 eV per reaction (Visible light flame)

$$1\,eV \sim 1.6 \times 10^{-12}\ erg/\text{reaction}$$

$$E^{chemical} \sim 1.6 \times 10^{-12}\ erg/\text{reaction} \cdot 6 \times 10^{55}\ \text{reactions} \sim 9.6 \times 10^{43}\ erg$$

$$E^{chemical} <<<< E_\odot^{life}\ !$$

Fusion!

$$\Delta E = \Delta m\, c^2$$

Some mass is converted into energy.

$$4\ {}^{1}p \longrightarrow 1\ {}^{4}He$$

4 protons converted into a Helium-4 nucleus
(2 protons, 2 neutrons)

$$4\, m(p) > m({}^{4}He) \Rightarrow \Delta m \sim 0.029 \text{ amu}$$

$$\Delta E = \Delta m \cdot c^2 \simeq \left(0.029 \text{ amu} \cdot 1.67 \times 10^{-24}\, g/\text{amu}\right) \cdot \left(3 \times 10^{10} \text{ cm/s}\right)^2$$

$$\sim 4.3 \times 10^{-5} \text{ erg} \cdot \frac{1 \text{ MeV}}{1.602 \times 10^{-6} \text{ erg}}$$

$$\sim 27 \text{ MeV}$$

About 0.7% of the mass of each proton is converted to energy by fusion!

$m(p) = 1.007276$ amu
$\frac{\Delta m}{4\, m(p)} \sim 0.007$

Assume entire Sun is made of protons:

$$E^{fusion} = 0.007 \cdot M_{\odot} \cdot c^2$$

$$= 0.007 \cdot \left(2 \times 10^{33}\, g\right) \cdot \left(3 \times 10^{10} \text{ cm/s}\right)^2$$

$$\sim 1.3 \times 10^{52} \text{ erg}$$

$$E^{fusion} > E_{\odot}^{life} \Rightarrow$$ Fusion can power the Sun!

The Sun burns about $M = \frac{L_{\odot}}{c^2} \sim 4.3 \times 10^{12}\, g \cdot s^{-1}$ of mass.