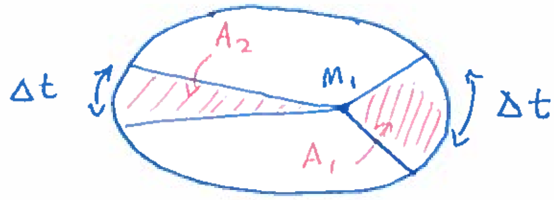
Kepler's 2^{ND} Law

ASTR 250 ①

The radius vector of the planet sweeps equal areas in equal amounts of time.

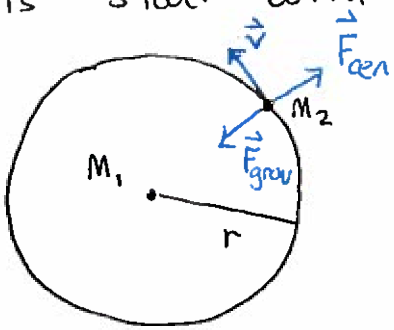


$A_1 = A_2$

⇒ Planet moves faster when closer to M_1

What is speed of orbiting planet?

Lets start with a circular orbit



$$\vec{F}_{cen} = \vec{F}_{grav}$$

$$\frac{\cancel{M_2} V^2}{r} = \frac{G M_1 \cancel{M_2}}{r^2}$$

$$V = \sqrt{\frac{GM_1}{r}} \sim \frac{1}{\sqrt{r}}$$

Keplerian velocities drop as $1/\sqrt{r}$!

In reality, planets move in elliptical orbits. The "Vis-Viva" equation gives you the velocity

$$V(r) = \sqrt{G(M_1+M_2)\left(\frac{2}{r} - \frac{1}{a}\right)}$$

Limits of this equation:

⊙ Circular orbits, when $(M_2 \ll M_1)$ and $a = r$ $(e=0)$ we get $V = \sqrt{GM_1\left(\frac{2}{r} - \frac{1}{r}\right)} = \sqrt{\frac{GM_1}{r}}$ Same equation as we derived before

⊙ $a \to \infty$ (UNBOUND)

$$\lim_{a\to\infty} V = \sqrt{GM_1\left(\frac{2}{r} - \frac{1}{\infty}\right)} \to \sqrt{\frac{2GM_1}{r}} = \sqrt{2} \cdot V_{circular}$$

$= $ Escape Speed

$$\frac{1}{2} M_2 V_{esc}^2 = \frac{GMM_2}{r} \Rightarrow V_{esc} = \sqrt{\frac{2GM_1}{r}}$$

$e = 1$ parabola $V = V_{esc}$

$e > 1$ hyperbola $V > V_{esc}$

Kepler's 3rd Law

$$P^2 = \frac{4\pi^2}{G(M_1+M_2)}\, a^3$$

Orbital Period

semi-major axis

UNITS: Let's look at the example of the Earth

$P = 1$ year $\quad a = 1$ AU $\quad M_2 \ll M_1$

$$P_{yrs}^2 = \frac{4\pi^2}{G \cdot (1 M_\odot)}\, a_{AU}^3$$

$$(1\,\text{yr})^2 = \frac{4\pi^2}{G(1 M_\odot)}\,(1\,\text{AU})^3$$

$$\Rightarrow G = 4\pi^2\ \text{AU}^3 \cdot \text{yrs}^{-2} \cdot M_\odot^{-1}$$

(Normal cgs units $\text{cm}^3 \cdot \text{s}^{-2} \cdot \text{g}^{-1}$)

Example: Halley's Comet $\quad a = 17.8$ AU

$$P_{yrs}^2 = (17.8\,\text{AU})^3$$

$$P_{yrs} = (17.8\,\text{AU})^{3/2} \sim 75.1\ \text{years}.$$

NOTE: This only works if $M_1 = 1 M_\odot$ and $M_2 \ll M_1$!