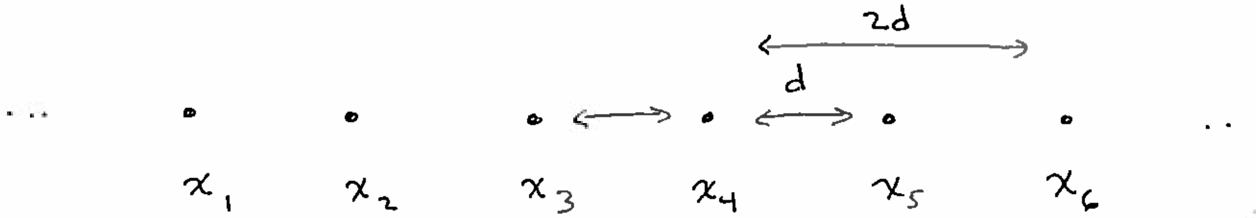
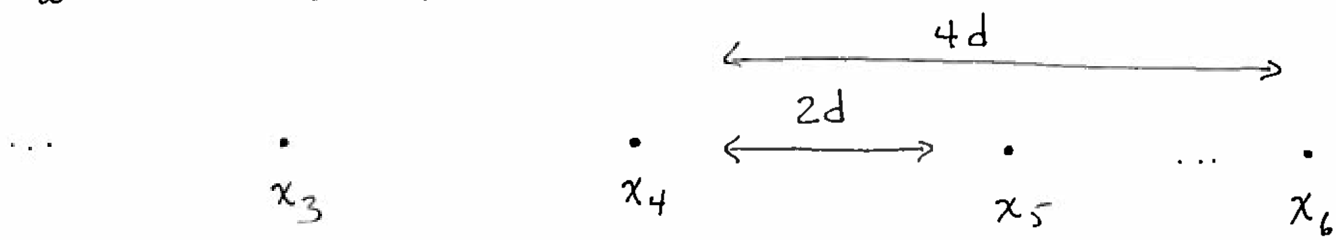
Hubble Law
Expansion of Spacetime

ASTR 250

Imagine of have a set of points ("co-moving coordinates")



If this 1D spacetime expands at a constant rate, then at some time later, the distance between points will have doubled:



The velocity of x_5 w.r.t. x_4 $\quad V_{5\text{-}4} = \frac{\Delta d}{\Delta t} = \frac{2d - d}{\Delta t} = \frac{d}{\Delta t}$

x_6 w.r.t x_4 $\quad V_{6\text{-}4} = \frac{\Delta d}{\Delta t} = \frac{4d - 2d}{\Delta t} = 2 \cdot \frac{d}{\Delta t}$

So $\boxed{V \sim d}$ the velocity of spacetime expansion is linearly proportional to distance away from x_4

This is the Hubble Law

NOTE: Choice of x_4 as center was arbitrary! There is NO center to the expansion. The problem is identical from point of view of x_3, x_5, x_6, etc!

For non-relativistic $V \sim c \cdot z$

Hubble Law $V = H_0 \cdot d$

Hubble Constant
subscript "0" means present epoch

Typical measurements give $H_0 \sim 70$ km/s/Mpc.

Problem: For $v \lesssim 0.3c$, $v \sim c \cdot z$ is a reasonable approximation. What distance does this correspond to?

$$d = \frac{0.3 \cdot 3 \times 10^5 \ \cancel{\mathrm{km/s}}}{70 \ \cancel{\mathrm{km/s}}/\mathrm{Mpc}} \sim 1290 \ \mathrm{Mpc}$$

For distances larger than this, should use cosmological model to calculate v or relationship between z and d.

Problem 2: Consider 2 galaxies separated by distance d and thus have $V_r = H_0 d$. If the relative speeds has been constant, then it took $t = \frac{d}{V_r} = \frac{d}{H_0 d} = \frac{1}{H_0}$

$$\frac{1}{H_0} = \frac{1}{70 \ \frac{\mathrm{km/s}}{\mathrm{Mpc}}} = \frac{1 \ \mathrm{Mpc}}{70 \ \frac{\mathrm{km}}{\mathrm{s}}} \times \frac{10^6 \ \mathrm{pc}}{\mathrm{Mpc}} \times \frac{3.08 \times 10^{13} \ \mathrm{km}}{1 \ \mathrm{pc}}$$

$$\sim 4.4 \times 10^{17} \ \mathrm{s} \sim 14 \ \mathrm{Gyrs}.$$

Rough estimate for age of Universe...