The LST depends on your position on the Earth!

Looking down on the Earth:

<table>
<thead>
<tr>
<th>Greenwich England</th>
<th>The difference between directions in space is just $L_w$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tucson, AZ</td>
<td></td>
</tr>
</tbody>
</table>

So \[ \text{LST} = \text{GST} + L_w \]

$t$  Local Sidereal Time in Greenwich, England.

$\text{GST}$ is tabulated in the Astronomical Almanac Section B.
Defn. of Great Circle: a circle on the surface of a sphere whose center is the center of the sphere.

Defn.: Spherical Triangle: a triangle on the surface of a sphere whose sides are made of 3 great circles.

Notice that the "sides" are really angles subtended from the center of the sphere.
Euclidean Geometry  
(Flat Space)  

Law of Cosines  
\[ a^2 = b^2 + c^2 - 2ab \cos \alpha \]  

Law of Sines  
\[ \frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c} \]  

Spherical Geometry  
(Non-Euclidean)  

Law of Cosines  
\[ \cos a = \cos b \cos c + \sin b \sin c \cos \alpha \]  

Law of Sines  
\[ \frac{\sin \alpha}{\sin a} = \frac{\sin \beta}{\sin b} = \frac{\sin \gamma}{\sin c} \]  

See online notes for a derivation of these equations or Fundamental Astronomy §2.1
**Example:** What is the HA of a source that is setting?

\[ NCP \]

\[ \phi \]

\[ S \]

\[ N \]

\[ W \]

\[ E \]

\[ NCP \]

\[ NCP \]

\[ \delta + 90^\circ \]

\[ \theta \]

\[ \phi \]

\[ \Psi \]

\[ HA \]

\[ 90^\circ - \delta \]

\[ 90^\circ \]

\[ 90^\circ - \phi \]

\[ Zenith \]

\[ Meridian \]

\[ our \ splecal \ triangle \]

Using the law of cosines:

\[
\cos 90^\circ = \cos(90 - \phi) \cos(90 - \delta) + \sin(90 - \phi) \sin(90 - \delta) \cos HA
\]

\[
90^\circ = \sin \phi \sin \delta + \cos \phi \cos \delta \cos HA
\]

\[
\cos HA = -\frac{\sin \phi \sin \delta}{\cos \phi \cos \delta} = -\tan \phi \tan \delta
\]

\[
HA = \cos^{-1} [-\tan \phi \tan \delta]
\]
Another Example: What is the angular separation between two stars with coordinates \((\alpha_1, \delta_1)\) and \((\alpha_2, \delta_2)\)?

\[
\begin{align*}
\cos \Delta \theta &= \cos (90 - \delta_1) \cos (90 - \delta_2) + \sin (90 - \delta_1) \sin (90 - \delta_2) \cos (\alpha_1 - \alpha_2) \\
\cos \Delta \theta &= \sin \delta_1 \sin \delta_2 + \cos \delta_1 \cos \delta_2 \cos (\alpha_1 - \alpha_2)
\end{align*}
\]

Let's do a numerical example: \((\alpha_1, \delta_1) = (10^\circ, -80^\circ)\) \((\alpha_2, \delta_2) = (11^\circ, 70^\circ)\)

\[
\alpha_2 - \alpha_1 = 11^\circ - 10^\circ = 1^\circ = \frac{360^\circ}{24^\circ} = 15^\circ
\]

So we have

\[
\cos \Delta \theta = \sin 80^\circ \sin 70^\circ + \cos 80^\circ \cos 70^\circ \cos 15^\circ
\]

\[\Rightarrow \Delta \theta = 10.6^\circ \quad \text{correct answer.}\]

For comparison, what if we used the Pythagorean theorem for plane triangles:

\[
\Delta \theta = \sqrt{(15^\circ)^2 + (10^\circ)^2} \quad \Rightarrow \quad \Delta \theta = 18^\circ \quad \text{incorrect!}
\]

Notice that the curvature on the sphere causes an error of \(\approx 4.4^\circ\) for even these closely spaced stars!
Epochs

Unfortunately, the Equatorial Coordinate system is not fixed.

\[ \text{The Earth precesses on its axis every } \sim 26,000 \text{ years} \Rightarrow \text{Direction of NCP slowly changes.} \]

\[ \Rightarrow \text{when you give coordinates } (\alpha, \delta), \text{ you must also give the epoch for when these coordinates are true!} \]

\[ \text{e.g. } 5^h \ 30^m \ 40.2^s \ +19^\circ \ 18' \ 33'' \ \text{ J2000.0 the epoch.} \]

Precession accounts for motion of the vernal equinox along the ecliptic (plane of the Earth's orbit) of \( \sim 50''/\text{year.} \)

**Aberration**

Imagine you have a telescope of length \( l \) moving at velocity \( \vec{V} \) with respect to a star.

In the time \( t = \frac{l}{c} \) it takes the light to traverse the telescope, the light beam will be displaced by \( \Delta x \).

\[ \Delta x = (\text{velocity component } \perp \text{ to light beam}) \cdot t \]
\[ = V \cdot \sin \theta \cdot t \]
\[ = V \cdot \sin \theta \cdot \frac{l}{c} \]

So, \( \alpha = \frac{\Delta x}{l} = \frac{V}{c} \cdot \sin \theta \)

The motion of the \( \sim 18 \text{km/s} \) Earth around the Sun accounts for up to 21'' in aberration angle!