(1) On April 20th, the brightest ever observed type Ia supernova explosion was reported in a host galaxy with a redshift \( z = 0.0736 \). If the maximum apparent magnitude was \( m_v = 17.1 \), then what was the maximum absolute magnitude, \( M_V \)? If the typical type Ia supernova has \( M_V = -19.3 \), then how many times brighter was this “over-luminous” supernova?

(2) The rate of change in the expansion of the universe is traditionally parametrized by \( q \), the “deceleration constant” (it is named “deceleration” because historically, an accelerating universe was considered unlikely!). The deceleration parameter is related to the Hubble constant by the first order differential equation

\[
1 + q = -\left(\frac{1}{H^2}\right) \frac{dH}{dt}
\]

Prove how \( q \) depends on the scale factor, \( a \), from the FRW metric. Hint: since it is a “deceleration” parameter, it probably has to depend on \( d^2a/dt^2 \).

(3) If \( q = 0 \), the universe is expanding at a constant rate. Then the age of the Universe is simply given by \( t = 1/H_0 \). What is this age in billions of years?

(4) Prove that the temperature of the Cosmic Microwave Background is warmer by a factor of \((1+z)\) at a redshift \( z \). Hint: Assume that the CMB is a blackbody for all \( z \) and use Wien’s Law. What was \( T_{cmb} \) at \( z = 6 \)?

(5) Submillimeter Galaxies (SMGs), are extremely dusty starburst galaxies that were discovered at high redshifts (\( z \sim 1 \) to 3). Assume the dust emission from SMGs are well characterized by blackbodies at a single dust temperature. If the observed spectrum of a SMG peaks at 180 \( \mu \)m, what would be its dust temperature if it is at a redshift of \( z = 2 \)?