

AST 300B – Spring 2017

In-class/take-home Problems Due: Monday April 17th

42. Hyperfine splitting of energy levels occurs because of the interactions between the spin of the nucleus and another angular momentum in the atom or molecule. The allowed resultant angular momentum states can be evaluated using a Clebsch-Gordan series.

(a) In the atomic case, the spin of the nucleus couples with the spin of the electron. The nuclear spin of ^1H is $I_{\text{nuc}} = \frac{1}{2}$ and the spin of the electron is $I_e = \frac{1}{2}$. What are the possible scalar values of the total spin (F) from the vector addition of the spin of the electron and the spin of the hydrogen nucleus? The hyperfine transition between these two states is observed at 21 cm or 1.4 GHz.

(b) The nuclear spin of ^2D is $I_{\text{nuc}} = 1$. What are the possible scalar values of the total spin (F) from the vector addition of the spin of the electron and the spin of the deuterium nucleus? The hyperfine transition between these two states is observed at 327 MHz.

(c) In the most common molecular case, the spin of the nucleus couples with the rotational angular momentum of the molecule (denoted J). Let's consider the HCN molecule where the nuclear spin of ^{14}N is $I_{\text{nuc}} = 1$ (ignore the spin of the H in this problem). What are the possible scalar values of the total angular momentum (also denoted F) from the vector addition of the spin of the nucleus and the rotational angular momentum of the molecules in the ground state $J = 0$ and in the first excited state $J = 1$? Given the selection rules for electric dipole transitions require that $\Delta F = 0$ (but NOT 0-0), +1, or -1, explain why the HCN $J = 1-0$ rotational transition has 3 spectral lines instead of just 1 spectral line (see Figure on back for an example HCN 1-0 spectrum taken at the ARO 12m telescope).

582; 2 L1521E HCN(1-0) 12M-MAC12 O: 26-MAR-2013 R: 26-MAR-2013
RA: 04:29:14.900 DEC: 26:13:56.60 (2000.0) Offs: 0.0 0.0 Eq
Unknown Tau: 0.1330 Tsys: 195.3 Time: 48.63 El: 45.76
N: 12288 IO: 6145. VO: -10.00 Dv: 4.1290E-02 LSR
FO: 88631.8470 Df: -1.2207E-02 Fi: 88631.8470
1274- 1278,

