

AST 300B – Spring 2018

In-class/take-home Problems Due: Friday April 13th

39. The emissivity coefficient j_ν ($\text{erg s}^{-1} \text{cm}^{-3} \text{ster}^{-1} \text{Hz}^{-1}$) for the two-photon transition $2s - 1s$ in the Hydrogen atom has the same equation as for an emission line except that the line profile function is replaced by $P_{2s}(\nu)$ which is the probability that a photon is emitted at frequency ν . The Einstein $A(2s-1s) \sim 8.23 \text{ s}^{-1}$. A Hydrogen atom energy level diagram is on the back.

- a. In the low density limit, collisional excitation and de-excitation are negligible, and the only way to populate the $2s$ level is by recombination at an effective recombination rate denoted by α_{2s}^{eff} . Write down the expression for j_ν in terms of $P_{2s}(\nu)$ and α_{2s}^{eff} assuming statistical equilibrium ($dn_{2s}/dt = 0$). [Hints: You will need to use statistical equilibrium to relate n_{2s} to α_{2s}^{eff} . What can you do with the Einstein B terms in the high ν limit?]
- b. It is possible that the density in HII regions can become high enough that the $2s$ level is de-excited by collisions with protons and electrons to the $2p$ level. The “downward” collision rates are given by $\gamma(2s-2p) = 4.74 \times 10^{-4} \text{ cm}^3 \text{ s}^{-1}$ for collisions with protons and by $\gamma(2s-2p) = 0.57 \times 10^{-4} \text{ cm}^3 \text{ s}^{-1}$ for collisions with electrons (Osterbrock 1974). What is the critical density ($n_{\text{crit}} \text{ cm}^{-3}$) for the $2s-1s$ transition?
- c. Given that the critical density in part (b) is not unreasonable for HII regions, we can't always assume that the low density limit applies. Modify your equation for j_ν assuming that collisional de-excitation by protons and electrons from $2s-2p$ cannot be ignored and assuming statistical equilibrium ($dn_{2s}/dt = 0$). [Hint: You don't need to worry about collisions from $2p-2s$ (because there is a **very fast** Einstein A from $2p-1s$) or between $2s$ and any other levels. Also, what can you assume about the number density of electrons and protons?]. Express your answer in terms of n_{crit} .

Energy Levels of Hydrogen ($n=1-4$)

Shells K ($n=1$,violet), L ($n=2$,blue), M ($n=3$,green) and N ($n=4$,red)

