Statistical Equilibrium Calculations

$$\frac{n_2}{n_1} = \frac{g_2}{g_1} e^{-h\nu/kT_{ex}} = \frac{B_{12} u_\nu + n_c \gamma_{12}}{A_{21} + B_{21} u_\nu + n_c \gamma_{21}}$$

IN General problems, can convert $\gamma_{12} \rightarrow \gamma_{21}$
and Einstein Bs $\Rightarrow$ Einstein As

$$\frac{\cancel{g_2}}{\cancel{g_1}} e^{-h\nu/kT_{ex}} = \frac{\cancel{\frac{g_2}{g_1}} \cdot \cancel{\frac{c^3}{8\pi h\nu^3}} A_{21} \cdot \cancel{\frac{8\pi h\nu^3}{c^3}} \frac{1}{e^{h\nu/kT_{bg}} - 1} + n_c \gamma_{21} \cancel{\frac{g_2}{g_1}} e^{-h\nu/kT_K}}{A_{21} + A_{21} \cdot \frac{1}{e^{h\nu/kT_{bg}} - 1} + n_c \gamma_{21}}$$

Define the (polarization averaged) photon occupation number

$$\bar{n}_\gamma = \frac{1}{e^{h\nu/kT_{bg}} - 1}$$

T_{bg} describes temperature of radiation field

Divide equation by A_{21} and define $n_{critical} = A_{21}/\gamma_{21}$ cm^{-3}

$$e^{-h\nu/kT_{ex}} = \frac{\bar{n}_\gamma + \frac{n_c}{n_{crit}} e^{-h\nu/kT_K}}{1 + \bar{n}_\gamma + \frac{n_c}{n_{crit}}}$$

As limits:
$n_c \rightarrow \infty$ $T_{ex} \rightarrow T_K$
$n_c \rightarrow 0$ $T_{ex} \rightarrow T_{bg}$