Collision Rates

ASTR 300B

Consider a 2-body collision between A and B

Reaction rate per unit volume $= n_A n_B \langle \sigma v \rangle_{AB} = n_A n_B \gamma$

where

$$\gamma = \langle \sigma v \rangle_{AB} = \int_0^\infty \sigma_{AB}(v)\, v\, f(v)\, dv \qquad cm^3 s^{-1}$$

For a thermal distribution at temperature T, a Maxwellian velocity distribution describes f(v):

$$f(v)\,dv = 4\pi \left(\frac{\mu}{2\pi kT}\right)^{3/2} e^{-\mu v^2/2kT}\, v^2\, dv$$

$$\mu = \frac{m_A m_B}{(m_A + m_B)} \text{ Reduced mass} \qquad E = \frac{\mu v^2}{2} \text{ is center-of-mass energy}$$

Sometimes more convenient to do problem in E rather than v

$$\gamma = \langle \sigma v \rangle_{AB} = \left(\frac{8kT}{\pi\mu}\right)^{1/2} \int_0^\infty \sigma_{AB}(E)\, \frac{E}{kT}\, e^{-E/kT}\, \frac{dE}{kT} \qquad cm^3 s^{-1}$$

Calculation of $\sigma_{AB}(v \text{ or } E)$ typically done for $u \to \ell \Rightarrow \gamma_{u\ell}$

For the special case of electron-ion inelastic scattering:

$$\langle \sigma v \rangle_{u\ell} \simeq \frac{8.629 \times 10^{-6}}{\sqrt{T}} \frac{\Omega_{u\ell}}{g_u} \qquad cm^3 s^{-1}$$

g_u: statistical weight

$\Omega_{u\ell}$: "Collision Strength" weak function of T that is near unity (~1-10) (takes care of quantum effects)