

Multi electron Atoms - Constructing a Term ASTR 300B

Electrons with orbital angular momentum \vec{l}_i have $(2l_i + 1)$ degenerate (unless there is a B/E-field present) energy levels.

Each electron also has a spin m_s . Pauli's exclusion principle forbids 2 electrons from having identical wavefunctions in the same orbital.

ONLY $2 \cdot (2l + 1) e^-$ allowed in each orbital
↑ due to m_s ↑ due to m_l

$l = 0$ s orbital $2 \cdot (2 \cdot 0 + 1) = 2 e^-$
 $\uparrow\downarrow = m_s = \pm 1/2$
 $m_l = 0$

$l = 1$ p orbital $2 \cdot (2 \cdot 1 + 1) = 6 e^-$
 $m_l = \frac{\uparrow\downarrow}{-1} \quad \frac{\uparrow\downarrow}{0} \quad \frac{\uparrow\downarrow}{+1} \leftarrow m_s = \pm 1/2$

$l = 2$ d orbital $2 \cdot (2 \cdot 2 + 1) = 10 e^-$
 $m_l = \frac{\uparrow\downarrow}{-2} \quad \frac{\uparrow\downarrow}{-1} \quad \frac{\uparrow\downarrow}{0} \quad \frac{\uparrow\downarrow}{+1} \quad \frac{\uparrow\downarrow}{+2} \leftarrow m_s = \pm 1/2$

The periodic table is organized by the energy of the orbitals (n, l)

↑ principle quantum number: i.e. 1s, 2s, 2p, etc.

Due to electromagnetic interactions, the e^- orbital angular momenta and spin couple. We use the rules of vector addition of quantized angular momenta.

For elements in the $\approx 1^{\text{st}}$ half of the periodic table, the "L-S Coupling" or Russell-Saunders Coupling best describes these interactions:

Each individual e^- has \vec{l}_i and \vec{s}_i

$$\vec{L} = \sum_i \vec{l}_i \quad M_L = \sum_i m_{l_i} \leftarrow \text{scalar values}$$

$$\vec{S} = \sum_i \vec{s}_i \quad M_S = \sum_i m_{s_i} \leftarrow \text{scalar values}$$

$$\vec{J} = \vec{L} + \vec{S} \quad \vec{J} \text{ is total angular momentum}$$

$$J = L+S, L+S-1, \dots, |L-S| \quad \text{Scalar values given by Glebsch-Gordon Series}$$

NOTE: We only have to really consider "valence electrons" in unfilled orbitals to construct the "term".

Spectroscopy: Atomic Terms Examples

HI $1s^1 \Rightarrow l_1 = 0 \Rightarrow L = 0 \Rightarrow S \text{ term}$
 $s_1 = 1/2 \Rightarrow S = 1/2 \Rightarrow 2S+1 = 2$
 $\Rightarrow J = 0 + 1/2 = 1/2$

Ground State
 TERM: $2S_{1/2}$

Note: this is the same answer for all Alkali Metals in the 1st column of periodic table since they all have valence electrons in a ns^1 configuration.

C II $(1s^2 2s^2) 2p^1 \Rightarrow l_1 = 1 = L = 1 \Rightarrow P \text{ term}$
 $s_1 = 1/2 \Rightarrow S = 1/2 \Rightarrow 2S+1 = 2$

$\Rightarrow J = 1 + 1/2, |1 - 1/2| = 3/2, 1/2$

Ground State
 TERMS: $2P_{1/2}$ $2P_{3/2}$ $\Rightarrow 2 \text{ energy levels!}$
 "Fine structure" Splitting

The transition between these levels is @ $158 \mu\text{m}$!

He I $1s^2 \Rightarrow l_1 = 0 \quad l_2 = 0 = L = 0 \Rightarrow S \text{ term}$
 $s_1 = 1/2 \quad s_2 = 1/2 \Rightarrow S = 1/2 + 1/2, |1/2 - 1/2| = 1, 0$

So $2S+1 = 3, 1$

$\Rightarrow J = 1+0, 0+0 = 1, 0$

Terms: $1S_0$ $3S_1$

NOTE: For $1s^2$ configs, Pauli Exclusion will eliminate the term, but it exists for $1s^2 2s^1$ etc.

Note that helium energy level diagram is usually organized into "singlet" terms & "triplet" terms.