Synchrotron Emission ASTR 300B

Typical spiral galaxy @ 1 GHz ~10% free-free and ~90% synchrotron
⇒ Synchrotron emission DOMINATES SED at low ν.

3 types of Magnetobremsstrahlung:

Gyro-radiation	$KE \ll m_e c^2$	$(v \lll c)$
Cyclotron	$KE \sim m_e c^2 \sim 511\,\text{keV}$	
Synchrotron	$KE \gg m_e c^2$	(typically very relativistic)

Lorentz Force is ⊥ to e^-/e^+ velocity ⇒ does not change $V_\parallel$ to B field:

$$\vec{F} = \frac{e(\vec{v} \times \vec{B})}{c} \qquad \text{so} \qquad m|\dot{v}| = m\omega^2 r = \frac{e}{c}\omega r B$$

Define gyro frequency: $$\omega_G \equiv 2\pi\nu_G \equiv \frac{eB}{mc}$$

$$\nu_G \simeq 2.8\ \text{MHz}\left(\frac{B}{1\,\text{gauss}}\right) \leftarrow$$

For typical ISM $B \sim 10\,\mu G$ ν_G too low to propagate far in ISM (plasma ν)

Synchrotron emission due to relativistic e^-/e^+ dominates.

Relativistic effects must now be accounted for:

$$E = \gamma m_e c^2 \qquad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \ \text{Lorentz factor}$$

In the rest frame of the e^-/e^+, the emitted Power is given by the Larmor formula but we must use Lorentz Transformations to the observer's frame

⇒ total power boosted by $\sim \gamma^2$

⇒ emission is beamed into narrow "cone" with $\theta \sim \frac{1}{\gamma}$

The formal derivation is very tedious. For instance, it can be shown that the Power Spectrum of a single e^-/e^+ is given by:

$$P(\nu) = \frac{\sqrt{3}\, e^2 B \sin\alpha}{m_e c^2} \left(\frac{\nu}{\nu_c}\right) \int_{\nu/\nu_c}^{\infty} K_{3/2}(x)\, dx$$

"pitch angle" w.r.t. B field

↑ modified Bessel fctn

Critical frequency $\nu_c \equiv \frac{3}{2} \gamma^2 \nu_G \sin\alpha \sim \gamma^2 \nu_G$

Synchrotron emission is typically <u>NON</u>-THE<u>RMA</u>L meaning the e^-/e^+ energies do <u>NOT</u> follow a Maxwell Boltzmann Distribution, but instead can be described by a power-law:

$$N(E) = N_0 E^{-\Gamma} \qquad \mathrm{cm}^{-3}\,\mathrm{erg}^{-1}$$

↑ this is just a constant

Total volume density
$N = \int_{E_{MIN}}^{E_{max}} N(E)\, dE$

If $\Gamma > 1/3$ the emissivity and absorption coefficients are:

$$j_\nu = c_5(\Gamma) N_0 B^{(\Gamma+1)/2} \left(\frac{\nu}{2c_1}\right)^{-\left(\frac{\Gamma-1}{2}\right)}$$

$$\alpha_\nu = c_6(\Gamma) N_0 B^{(\Gamma+2)/2} \left(\frac{\nu}{2c_1}\right)^{-\left(\frac{\Gamma+4}{2}\right)}$$

important for synchrotron self-absorption

where c_1, c_5, c_6 are Pacholczyk's Constants

$$c_1 = 6.27 \times 10^{18}$$

Many sources in the galaxy have $\Gamma \sim 2.5$