LIMITS OF RADIATIVE TRANSFER Equation

ASTR 300B

Review:

Emissivity coefficient $\mathrm{erg \cdot s^{-1} \cdot cm^{-3} \cdot ster^{-1} \cdot Hz^{-1}}$

$$\frac{dI_\nu}{ds} = j_\nu - \alpha_\nu I_\nu$$

absorption coefficient $\mathrm{cm^{-1}}$

Source Function $S_\nu \equiv j_\nu / \alpha_\nu$ $\mathrm{erg \cdot s^{-1} \cdot cm^{-2} \cdot ster^{-1} \cdot Hz^{-1}}$

$$\frac{dI_\nu}{d\tau_\nu} = S_\nu - I_\nu$$

optical depth $d\tau_\nu = \alpha_\nu \, ds$ unitless

"Thermal" emission $S_\nu = B_\nu(T)$

$B_\nu(T) = j_\nu / \alpha_\nu$ Kirchoff's Law

Formal 1D Solution to Radiative Transfer Equation

If S_ν = const:

$$I_\nu = I_\nu(0)\, e^{-\tau_\nu} + S_\nu\left(1 - e^{-\tau_\nu}\right)$$

background intensity

① Consider the limit - Optically thick $\tau_\nu \gg 1 \Rightarrow e^{-\tau_\nu} \to 0$

$$\lim_{\tau_\nu \gg 1} I_\nu = I_\nu(0)\, \cancel{e^{-\tau_\nu}}^{\,0} + S_\nu\left(1 - \cancel{e^{-\tau_\nu}}^{\,0}\right) = S_\nu$$

In optically thick limit $\underline{I_\nu = S_\nu}$

② Consider the limit - Optically thin $\tau_\nu \ll 1 \Rightarrow$ expand $e^{-\tau_\nu}$ in Taylor Series $\left\{ e^{-\tau_\nu} \simeq 1 - \tau_\nu \right.$

$$I_\nu = I_\nu(0)\left(1 - \tau_\nu\right) + S_\nu\left[1 - \left(1 - \tau_\nu\right)\right]$$

$$I_\nu = I_\nu(0)\left(1 - \tau_\nu\right) + S_\nu \cdot \tau_\nu$$

If background negligible: $\underline{I_\nu = S_\nu \tau_\nu}$

Remember: $\tau_\nu = \int \alpha_\nu \, ds$