Blackbody Radiation

A blackbody does not reflect or scatter light but absorbs and emits light completely.

Planck's Law describes the specific intensity of a blackbody:

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \cdot \frac{1}{e^{h\nu/kT}-1} \qquad \mathrm{erg \cdot s^{-1} \cdot cm^{-2} \cdot ster^{-1} \cdot Hz^{-1}}$$

we can relate this to $B_\lambda(T)$:

$$B_\lambda(T) = \frac{c}{\lambda^2} B_\nu(T)$$

$$= \frac{c}{\lambda} \cdot \frac{2h}{c^2} \cdot \left(\frac{c^3}{\lambda^3}\right) \cdot \frac{1}{e^{hc/\lambda kT} - 1}$$

$$= \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT}-1} \qquad \mathrm{erg \cdot s^{-1} \cdot cm^{-2} \cdot ster^{-1} \cdot cm^{-1}}$$

The total intensity of a blackbody is:

$$B(T) = \int_0^\infty B_\nu(T)\, d\nu = \frac{2h}{c^2} \int_0^\infty \frac{\nu^3\, d\nu}{e^{h\nu/kT}-1}$$

Let $x = h\nu/kT \Rightarrow \nu = \frac{kT}{h} \cdot x \Rightarrow d\nu = \frac{kT}{h} dx$

$$B(T) = \frac{2h}{c^2} \cdot \left(\frac{kT}{h}\right)^4 \int_0^\infty \frac{x^3 dx}{e^x - 1}$$

← This integral is $\int_0^\infty \frac{x^a dx}{e^x-1} = \underbrace{\Gamma(a+1)}_{\text{Gamma Fctn.}} \underbrace{\zeta(a+1)}_{\text{Reimann Zeta Fctn.}}$

$$B(T) = \frac{2}{c^2} \cdot \frac{k^4 T^4}{h^3} \cdot \Gamma(4) \cdot \zeta(4) = \frac{2}{c^2} \frac{k^4 T^4}{h^3} \cdot 6 \cdot \frac{\pi^4}{90}$$

$$= \frac{2\pi^4 k^4}{15 c^2 h^3} T^4$$

← NOTICE T^4 dependence

$\mathrm{erg \cdot s^{-1} \cdot cm^{-2} \cdot ster^{-1}}$

Emergent Flux (or "astrophysical flux") of a black body is:

$$F^+ = \pi \cdot B(T) = \pi \cdot \frac{2\pi^4 k^4}{15\, c^2 h^3}\, T^4$$

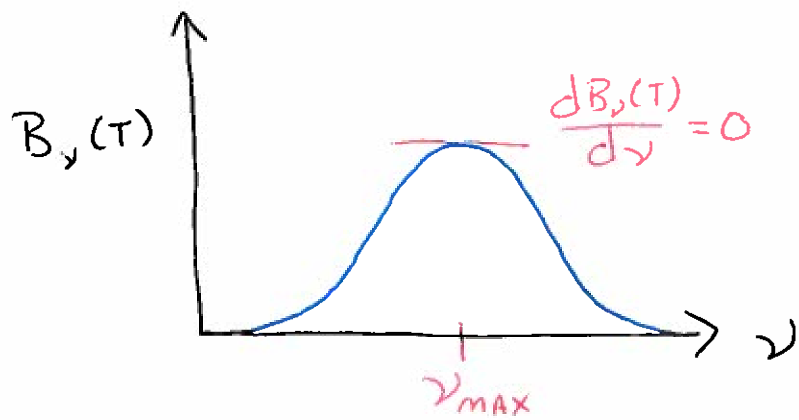
$$= \sigma\, T^4 \quad erg \cdot s^{-1} \cdot cm^{-2}$$

σ = Stephan - Boltzmann Constant $\sim 5.67 \times 10^{-5}\ erg \cdot s^{-1} \cdot cm^{-2} \cdot K^{-4}$

For a star ($\approx$ a good blackbody!):

$$L_* = 4\pi R_*^2 F_*^+ = 4\pi R_*^2\, \sigma\, T_*^4$$

Let plot the Planck Function:



The Planck function goes to zero at the limits $\to 0$ and $\to \infty$ and has a single maximum. We have to use numerical techniques to find the maximum:

$$\frac{dB_\nu(T)}{d\nu} = 0 \implies \frac{\nu_{MAX}}{T} = 5.879 \times 10^{10}\ Hz \cdot K^{-1}$$

$$\frac{dB_\lambda(T)}{d\nu} = 0 \implies \lambda_{MAX}\, T = 0.2898\ cm \cdot K$$

Wein's Displacement Law

LIMITS of the Planck Function

Notice that $h\nu/k$ has units of T (kelvin)

① High ν limit: $\frac{h\nu}{kT} \gg 1$ Wein Approximation

$$\lim_{\nu \to \text{large}} B_\nu(T) \longrightarrow e^{-h\nu/kT}$$

exponential dependence at high ν

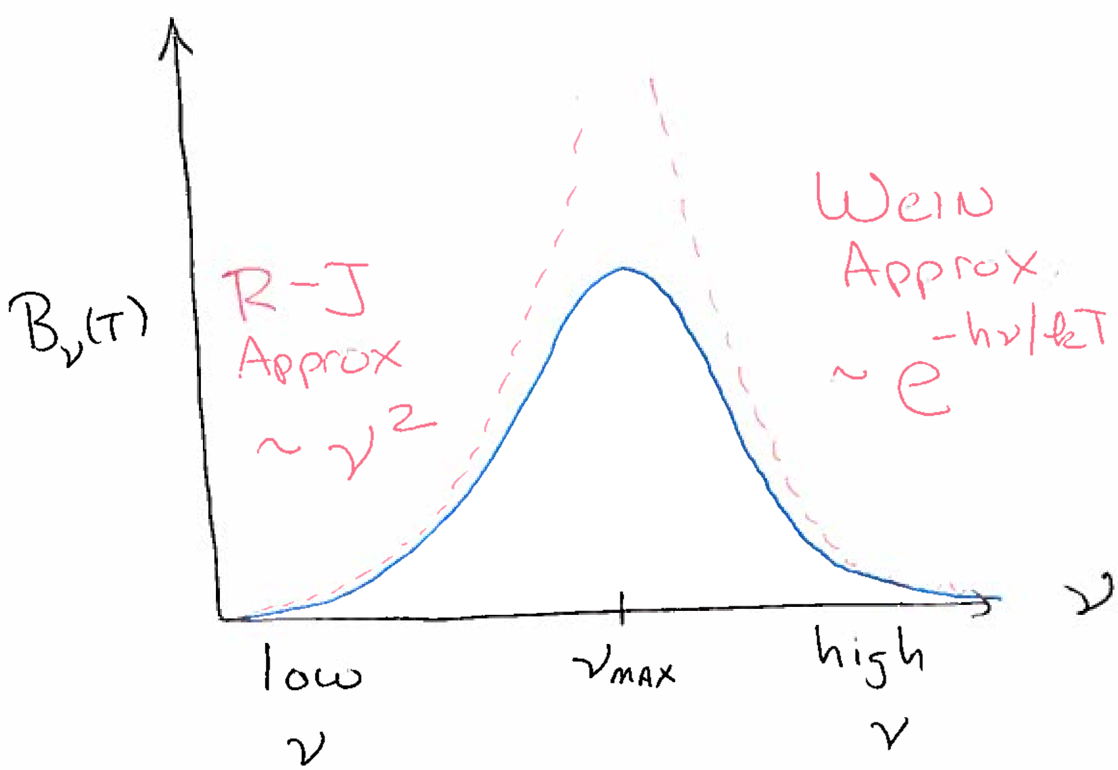
② Low ν limit: $\frac{h\nu}{kT} \ll 1$ Rayleigh-Jeans Approximation

Taylor expand: $e^{h\nu/kT} \simeq 1 + h\nu/kT$

$$\lim_{\nu \to \text{small}} B_\nu(T) = \frac{2h\nu^3}{c^2} \cdot \frac{1}{\cancel{1} + \frac{h\nu}{kT} - \cancel{1}} = \frac{2kT\nu^2}{c^2}$$

Intensity linearly proportional to T

Notice ν^2 dependence at low ν



Energy Density of a Blackbody

$$u_\nu = \frac{1}{c} \oint B_\nu(T)\, d\Omega$$

Assuming $B_\nu(\theta, \phi) = \text{const}$

$$u_\nu = \frac{4\pi}{c} \cdot \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT}-1} = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/kT}-1}$$

$erg \cdot cm^{-3} \cdot Hz^{-1}$

The total energy density is given by:

$$u = \frac{4\pi}{c} \cdot \int_0^\infty B_\nu(T)\, d\nu = \frac{4\cancel{\pi}}{c} \cdot \frac{\sigma T^4}{\cancel{\pi}}$$

$$= \frac{4\sigma}{c} T^4$$

$$= a T^4 \qquad erg \cdot cm^{-3}$$

called the radiation constant

$a = 7.567 \times 10^{-15}\ erg \cdot cm^{-3} \cdot K^{-4}$