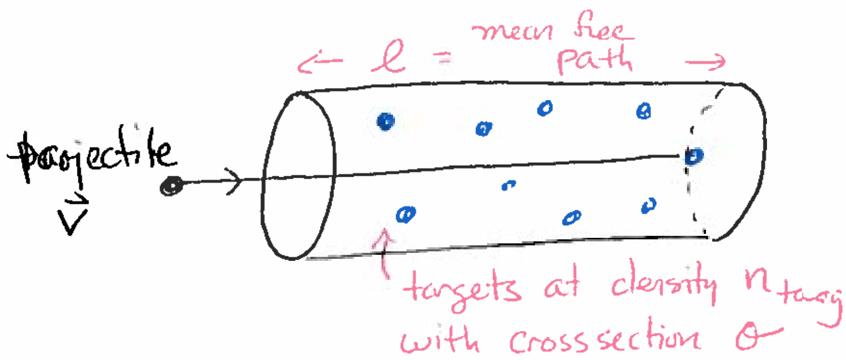
Photoionization & Recombination

ASTR 300B



Mean Free Path : $\ell \sim \dfrac{1}{n_{targ}\,\sigma}$ cm

$\quad\quad$ $cm^{-3}\ cm^{2}$

Time between Collisions : $\sim \dfrac{1}{n_t \sigma v}$ s

$\quad\quad$ $cm^{-3}\ cm^{2}\cdot \frac{cm}{s}$

Collision Rate : $\sim n_t \sigma v$ s^{-1}

Collision Rate per unit volume : $\sim n_{proj.}\, n_{targ.}\, \sigma v$ $cm^{-3}\ s^{-1}$

Now, in a thermal gas, the projectiles are not moving at a single velocity v, but instead have a distribution of velocities or energies. Furthermore, for processes like photoionization or recombination, the cross section depends on energy. Thus we must integrate $\langle \sigma v \rangle$ over the energy of particles:

Sometimes called a "Collision Rate Coefficient"

$$\langle \sigma v \rangle = \int_0^\infty \sigma(E)\, v\, f(E)\, dE \qquad cm^3 \cdot s^{-1}$$

$cm^2 \cdot \frac{cm}{s}$

for a thermal gas this is the Maxwell-Boltzmann Distribution

Let's start with photoionization rate $\left(\begin{array}{l}\text{a collision rate} \\ \text{for photons}\end{array}\right)$

$$\zeta_{pi} = \int_{\nu_I}^{\infty} \sigma_{pi}(\nu) \; c \; \frac{u_\nu}{h\nu} \, d\nu \qquad s^{-1}$$

ionization energy of atom or molecule

cm^2

photon velocity $\frac{cm}{s}$

Number of photons at $h\nu$ per cm^3

$\frac{erg \cdot cm^{-3}\, Hz^{-1}}{erg} \cdot Hz$

For hydrogenic atoms (ions) $\sigma_{pi}(\nu)$ may be approximated as a power-law when $Z^2 I_H < h\nu \lesssim 100 Z^2 I_H$

$$\sigma_{pi}(\nu) \sim (6.304 \times 10^{-18} Z^{-2}) \left(\frac{h\nu}{Z^2 I_H}\right)^{-3} \qquad cm^2$$

At higher energies, the power-law steepens to -3.5 until $h\nu \simeq 2.5$ keV where it becomes equal to the Compton Scattering cross section

How many IONIZING PHOTONS with $h\nu > 13.6$ eV are produced by a star per second?

$$Q_0 = 4\pi R_*^2 \int_{\nu_I = 13.6eV}^{\infty} \frac{\pi B_\nu(T)}{h\nu} \, d\nu \qquad s^{-1}$$

cm^2 $\quad erg \cdot s^{-1} \cdot cm^{-2} \cdot Hz^{-1}$ $\quad erg$ $\quad Hz$

Can lookup Q_0 for different types of stars in tables

Also define Q_1 as same integral but for $h\nu_I = 24.6$ eV capable of ionizing Helium.

Now let's consider recombination. The recombination rate is an example of a $\langle \sigma v \rangle$ collisional rate coefficient:

$$\underbrace{\alpha_{rr}}_{\text{radiative recombination}}(T) = \underbrace{\langle \sigma v \rangle}_{cm^3 \cdot s^{-1}}$$

integrated over Maxwell-Boltzmann Energy Distribution

Let's do a problem to illustrate:

In HI regions, Carbon can be ionized because the ionization energy of C is 11.26 eV < 13.6 eV need to ionize H. The typical $n_e \sim 0.04\ cm^{-3}$ and $T \sim 100$ K. Balancing photoionization and recombination, we can derive the fraction of neutral Carbon atoms in a HI cloud:

Recombination Collison rate per unit volume = Ionization Collision rate per unit volume

$$\underbrace{\alpha_{rr}(C^+)}_{cm^3 \cdot s^{-1}} \cdot \underbrace{n_e}_{cm^{-3}} \cdot \underbrace{n_{C^+}}_{cm^{-3}} = \underbrace{\zeta(C)}_{s^{-1}} \cdot \underbrace{n_C}_{cm^{-3}}$$

Divide by $n_{C^+} + n_C$:

$$\alpha_{rr}(C^+)\, n_e \underbrace{\frac{n_{C^+}}{n_{C^+} + n_C}}_{1 - \text{fraction of neutral Carbon}} = \zeta(C) \cdot \underbrace{\frac{n_C}{n_{C^+} + n_C}}_{\text{fraction of neutral Carbon}}$$

$$\alpha_{rr}(C^+)\, n_e \left[1 - f(C)\right] = \zeta(C) \cdot f(C)$$

$$f(C) = \frac{\alpha_{rr}(C^+)\, n_e}{\alpha_{rr}(C^+)\, n_e + \zeta(C)}$$

Looking up rates in Draine: Table 13.1 $\zeta(C) \sim 2.58 \times 10^{-10}\ s^{-1}$

Table 14.6 $\alpha_{rr}(C^+) \sim 8.63 \times 10^{-12}\ cm^3 \cdot s^{-1}$

$f(C) \sim 1.3 \times 10^{-3}$ $\Rightarrow$ 99.9% of Carbon is IONIZED in HI Clouds!