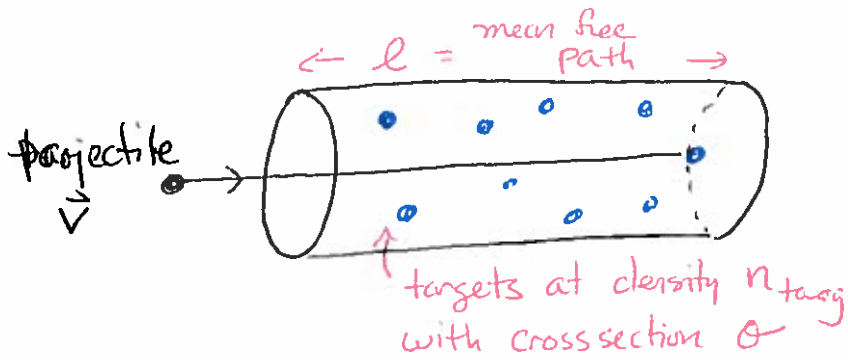


Photoionization & Recombination

ASTR
300B



Mean Free Path \circ $l \sim \frac{1}{n_{\text{targ}} \sigma}$ cm
 $\text{cm}^{-3} \text{cm}^2$

Time between Collisions \circ $\sim \frac{1}{n_{\text{targ}} \sigma v}$ s
 $\text{cm}^{-3} \text{cm}^2 \cdot \frac{\text{cm}}{\text{s}}$

Collision Rate \circ $\sim n_{\text{targ}} \sigma v$ s^{-1}

Collision Rate per unit volume \circ $\sim n_{\text{proj}} \cdot n_{\text{targ}} \cdot \sigma v$ $\text{cm}^{-3} \text{s}^{-1}$

Now, in a thermal gas, the projectiles ^{are} not moving at a single velocity v , but instead have a distribution of velocities or energies. Furthermore, for processes like photoionization or recombination, the cross section depends on energy. Thus we must integrate $\langle \sigma v \rangle$ over the energy of particles \circ

Sometimes called a "Collision Rate Coefficient"

$$\langle \sigma v \rangle = \int_0^{\infty} \sigma(E) v f(E) dE \quad \text{cm}^3 \cdot \text{s}^{-1}$$

for a thermal gas this is the Maxwell-Boltzmann Distribution

Let's start with photoionization rate (a collision rate) (for photons)

$$\Sigma_{pi} = \int_{\nu_I}^{\infty} \sigma_{pi}(\nu) c \frac{2\nu}{h\nu} d\nu \quad S^{-1}$$

ionization energy of atom or molecule $\rightarrow \nu_I$
 cm^2
 photon velocity $\frac{cm}{s}$
 Number of photons at $h\nu$ per cm^3
 $\frac{erg \cdot cm^{-3} Hz^{-1}}{erg} \cdot Hz$

For hydrogenic atoms (ions) $\sigma_{pi}(\nu)$ may be approximated as a power-law when $Z^2 I_H < h\nu \lesssim 100 Z^2 I_H$

$$\sigma_{pi}(\nu) \sim (6.304 \times 10^{-18} Z^{-2}) \left(\frac{h\nu}{Z^2 I_H} \right)^{-3} \quad cm^2$$

At higher energies, the power-law steepens to -3.5 until $h\nu \approx 2.5 keV$ where it becomes equal to the Compton scattering cross section

How many ionizing photons with $h\nu > 13.6 eV$ are produced by a star per second?

$$Q_0 = 4\pi R_*^2 \int_{\nu_I = 13.6 eV}^{\infty} \frac{\pi B_\nu(T)}{h\nu} d\nu \quad S^{-1}$$

cm^2
 $erg \cdot s^{-1} \cdot cm^{-2} \cdot Hz^{-1}$
 erg
 Hz

Can lookup Q_0 for different types of stars in tables

Also define Q_1 as same integral but for

$h\nu_I = 24.6 eV$ capable of ionizing Helium.

Now let's consider recombination. The recombination rate is an example of a $\langle \sigma v \rangle$ collisional rate coefficient:

$$\alpha_{rr}(T) = \langle \sigma v \rangle \quad \text{integrated over Maxwell-Boltzmann Energy Distribution}$$

$\text{cm}^3 \cdot \text{s}^{-1}$

radiative recombination \rightarrow

Let's do a problem to illustrate:

In HI regions, Carbon can be ionized because the ionization energy of C is $11.86 \text{ eV} < 13.6 \text{ eV}$ need to ionize H.

The typical $n_e \sim 0.04 \text{ cm}^{-3}$ and $T \sim 100 \text{ K}$. Balancing photoionization and recombination, we can derive the fraction of neutral carbon atoms in a HI cloud:

Collision rate per unit volume (Recombination) = Collision rate per unit volume (Ionization)

$$\alpha_{rr}(C^+) \cdot n_e \cdot n_{C^+} = \zeta(C) \cdot n_C$$

$\text{cm}^3 \cdot \text{s}^{-1} \cdot \text{cm}^{-3} \cdot \text{cm}^{-3} = \text{s}^{-1} \cdot \text{cm}^{-3}$

Divide by $n_{C^+} + n_C$

$$\alpha_{rr}(C^+) n_e \frac{n_{C^+}}{n_{C^+} + n_C} = \zeta(C) \cdot \frac{n_C}{n_{C^+} + n_C}$$

fraction of neutral carbon

1 - fraction of neutral Carbon \rightarrow

$$\alpha_{rr}(C^+) n_e [1 - f(C)] = \zeta(C) \cdot f(C)$$

$$f(C) = \frac{\alpha_{rr}(C^+) n_e}{\alpha_{rr}(C^+) n_e + \zeta(C)}$$

Looking up rates in Draine: Table 13.1 $\zeta(C) \sim 2.58 \times 10^{-10} \text{ s}^{-1}$

Table 14.6 $\alpha_{rr}(C^+) \sim 8.63 \times 10^{-12} \text{ cm}^3 \cdot \text{s}^{-1}$

$$f(C) \sim 1.3 \times 10^{-3} \Rightarrow 99.9\% \text{ of Carbon is IONIZED in HI clouds!}$$

Let's consider a HII Region :

Assume Steady State / Equilibrium \Rightarrow

Number of Ionizations per second = Number of Recombinations per second

$$Q_0 \text{ s}^{-1} = \alpha_{rr}(T) n_e n_i \cdot \text{Volume of HII Region cm}^3$$

$\text{cm}^3 \cdot \text{s}^{-1} \cdot \text{cm}^{-3} \cdot \text{cm}^3 = \text{cm}^3$

Sometimes a simple assumption that all e^- come from H can be made and $n_e = n_i$

$$Q_0 = \alpha_{rr}(T) n_e^2 \frac{4}{3} \pi R_{\text{HII}}^3$$

R_{HII} is called the Strömgren radius

$$R_{\text{HII}} = \left(\frac{3}{4\pi} \frac{Q_0}{\alpha_{rr}(T) n_e^2} \right)^{1/3} \text{ cm}$$

For HII regions, the correct recombination coefficient is $\alpha_B(T)$ for Hydrogen.

"CASE B Recombination". We will discuss this in detail next week ...