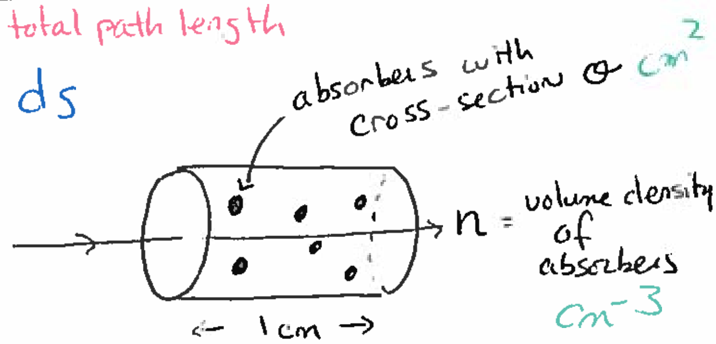
Optical Depth & Absorption Coefficient

ASTR 300B

$$\tau_\nu \equiv \int_0^L \alpha_\nu \, ds$$ ← total path length

$\alpha_\nu \sim$ # of photon absorptions per cm^{-1}



$$\alpha_\nu = n \cdot \sigma_\nu \qquad cm^{-1}$$

$$cm^{-3} \cdot cm^{2}$$

So really τ_ν is given by

$$\tau_\nu = \int_0^L n \, \sigma_\nu \, ds = \sigma_\nu \int_0^L n \, ds$$

If σ_ν independent of s

This is the Column Density

$$N \equiv \int n \, ds \qquad cm^{-2}$$

$$cm^{-3} \cdot cm$$

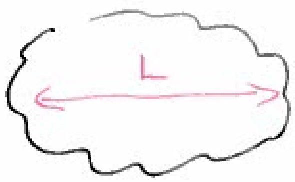
Column Density is a measure of how much "stuff" you have along a line of sight (in a particular direction).

$$\tau_\nu = \sigma_\nu \cdot N$$

Optical depth = cross-section of absorbers × Column Density of absorber

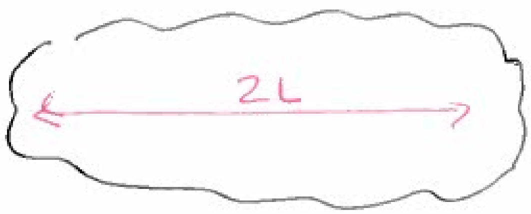
Practical Example:

Assume you have 2 clouds with absorbers with cross-sections σ_ν (same in both clouds) and n same in both clouds.



Cloud 1

$$N_1 = \int_0^L n\,ds = n \cdot L$$



Cloud 2

$$N_2 = \int_0^{2L} n\,ds = 2nL$$

$$\frac{\tau_\nu(\text{cloud 2})}{\tau_\nu(\text{cloud 1})} = \frac{2\sigma_\nu \cdot n \cdot L}{\sigma_\nu \cdot n \cdot L} = 2$$

Optical depth scales linearly with how much "stuff" you have along a line of sight.

Mean-free Path Length

From the definition of optical depth:

$$\tau_\nu = \int_0^L \alpha_\nu \, ds = \alpha_\nu \cdot L$$

if α_ν independent of s.

When $\tau = 1$:

$$1 = \alpha_\nu \cdot L$$

$$\Rightarrow L = \frac{1}{\alpha_\nu} = \frac{1}{n\sigma_\nu}$$

← this is the mean free path length of a photon

$\tau = 1$ occurs at the mean free path length