Equilibrium Dust Temperature

Balance radiative heating and cooling of (large) dust grains (ignoring gas collisional heating, cosmic ray impact heating etc.)

Equilibrium (LTE) $\Rightarrow$ $\left(\frac{dE}{dt}\right)_{emit} = \left(\frac{dE}{dt}\right)_{absorb}$

* = Volume ~ size of grain

Consider an optically thin emitting volume* with 1 dust grain:

$$L_\nu = 4\pi j_\nu V = 4\pi \alpha_\nu B_\nu(T_d) \cdot V = 4\pi \cdot \pi a^2 Q_{abs}(\nu) B_\nu(T_d) \cancel{n_d} \cdot V$$

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$$\left(\frac{dE}{dt}\right)_{emit} = \int_0^\infty L_\nu \, d\nu = \int_0^\infty 4\pi \cdot \pi a^2 Q_{abs}(\nu) \, B_\nu(T_d) \, d\nu$$

$erg \cdot s^{-1}$ $\qquad$ ster $\cdot$ cm^2 $\cdot$ $erg \cdot s^{-1} \cdot cm^{-2} \cdot ster^{-1} \cdot Hz^{-1}$ $\cdot Hz$

We define a "Planck weighted" Q_{abs}

$$\langle Q_{abs} \rangle_{T_d} \equiv \frac{\int_0^\infty B_\nu(T_d) Q_{abs}(\nu) \, d\nu}{\int_0^\infty B_\nu(T_d) \, d\nu}$$

Then

$$\left(\frac{dE}{dt}\right)_{emit} = 4\pi a^2 \cdot \pi \int_0^\infty B_\nu(T_d) d\nu \cdot \langle Q_{abs} \rangle_{T_d}$$

$$= 4\pi a^2 \cdot \sigma T_d^4 \cdot \langle Q_{abs} \rangle_{T_d}$$

At long λ (typically $\gtrsim 100\,\mu m$) opacities vary as power-law:

$$Q_{abs}(\nu) = Q_0 \left(\frac{\nu}{\nu_0}\right)^\beta$$

$$\langle Q_{abs} \rangle_{T_d} = \frac{\frac{2h}{c^2} \Gamma(4+\beta) \zeta(4+\beta) \frac{Q_0}{\nu_0^\beta} \left(\frac{kT_d}{h}\right)^{4+\beta}}{\frac{\sigma T_d^4}{\pi}} \sim T_d^\beta$$

$$\left(\frac{dE}{dt}\right)_{emit} \sim T_d^{4+\beta}$$

Now let's consider absorption:

$$\left(\frac{dE}{dt}\right)_{abs} = \int_0^\infty \underbrace{F_\nu}_{\substack{\text{Incident}\\ \text{flux density}}} \ \underbrace{\pi a^2 Q_{abs}(\nu)}_{\substack{\text{effective grain}\\ \text{cross section}}} \, d\nu$$

$erg \cdot s^{-1}$ $\qquad erg \cdot s^{-1} \cdot cm^{-2} \cdot Hz^{-1}$ $\qquad cm^2$ $\qquad Hz$

If the incident flux density comes from small discreet sources $F_\nu \sim c \cdot u_\nu$

Another way to see this: $\frac{u_\nu}{h\nu} \cdot c \cdot h\nu$

# of photons per cm^{-3} per Hz — moving at c — with energy $h\nu$

$cm^{-3} \cdot Hz^{-1}$ $\quad cm \cdot s^{-1}$ $\quad \cdot erg$

$\Rightarrow erg \cdot s^{-1} \cdot cm^{-2} \cdot Hz^{-1}$ ✓

If we define $\langle Q_{abs} \rangle_{u_*} \equiv \dfrac{\int_0^\infty u_\nu Q_{abs}(\nu) d\nu}{\int_0^\infty u_\nu d\nu}$ ← total energy density $= u_*$

Then $\left(\frac{dE}{dt}\right)_{abs} = \pi a^2 \langle Q_{abs} \rangle_{u_*} \cdot u_* \cdot c$

For the Interstellar Radiation Field (ISRF):

$u_* = 1.05 \times 10^{-12} \cdot U$ $erg \cdot cm^{-3}$ $\qquad U = 1$ is the average ISRF from Mathis et al. 1983

For Grains in Equilibrium:

$$4\pi a^2 \langle Q_{abs} \rangle_{T_d} \sigma T_d^4 = \pi a^2 \langle Q_{abs} \rangle_{u_*} u_* c$$

For Dust with $\beta = 2$ $\left(\langle Q_{abs} \rangle_{T_d} \sim T_d^\beta \sim T_d^2 \right)$

$$T_d \simeq u_*^{1/6}$$

Weak function of energy density

$\Rightarrow$ Double u_* → T_d increases by 12%

10 × u_* → T_d increases by 47%

100 × u_* → T_d increases by factor of 2.15

Empirical Dust temperature (Draine Ch. 24):

For silicate dust grains:

$$\langle Q_{abs} \rangle_{T_d} \simeq 1.3 \times 10^{-6} \left(\frac{a}{0.1 \mu m} \right) \left(\frac{T_d}{1 K} \right)^2$$

$$\langle Q_{abs} \rangle_{ISRF=1} \simeq 0.18 \left(\frac{a}{0.1 \mu m} \right)^{3/5} \qquad 0.01 \mu m \lesssim a \lesssim 1 \mu m$$

$u = 1$ "Standard" ISRF

In Equilibrium:

$$T_d = \left(\frac{\langle Q_{abs} \rangle_{ISRF=1} \cdot 1.05 \times 10^{-12} \cdot c}{4 \sigma \; 1.3 \times 10^{-6} \left(\frac{a}{0.1 \mu m} \right)} \right)^{1/6}$$

$T_d \simeq 16.4$ K for $a \sim 0.1 \mu m$ grains (weak function of grain size) $T_d \sim a^{-1/15}$

Dust grains are cold in average ISM!