Optical Depth + Column Density

ASTR 300B

Consider the case of pure emission with a background $I_\nu(0)$: (optically thin)

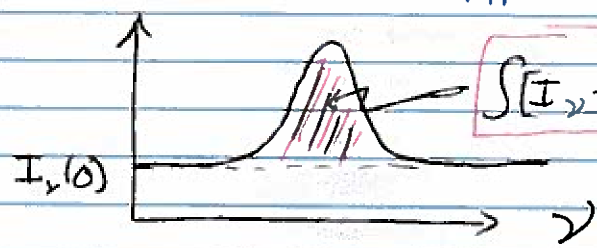
$$I_\nu = I_\nu(0) + \int j_\nu \, ds$$

$$= I_\nu(0) + \int \frac{h\nu}{4\pi} A_{ul} n_u \phi_\nu \, ds$$

Subtracting off the background:

$$I_\nu - I_\nu(0) = \frac{h\nu}{4\pi} A_{ul} \phi_\nu \underbrace{\int n_u \, ds}_{N_u}$$

N_u column density in upper state



$\boxed{\int [I_\nu - I_\nu(0)] \, d\nu}$ INTEGRATED INTENSITY

$$\int I_\nu - I_\nu(0) \, d\nu = \frac{h\nu}{4\pi} A_{ul} N_u$$

Remember $\int \phi_\nu \, d\nu = 1$ by definition

$erg \cdot s^{-1} \cdot cm^{-2} \cdot ster^{-1}$ $\quad$ $erg \cdot ster^{-1} \cdot s^{-1} \cdot cm^{-2}$ ✓

Integrated Intensity $\sim$ Column Density (in upper state)

optically thin emission

You can then use Boltzmann to get total column density

Since $\dfrac{n_u}{n_{TOT}} = \dfrac{g_u}{Q(T_{ex})} e^{-E_u/k T_{ex}}$ with $Q(T_{ex}) = \sum_i g_i e^{-E_i/k T_{ex}}$

And since $\int n \, ds \equiv N \Rightarrow \dfrac{N_u}{N_{TOT}} = \dfrac{g_u}{Q(T_{ex})} e^{-E_u/k T_{ex}}$

But what about optical depth?

Remember definition $\tau_\nu = \int \alpha_\nu \, ds$

$$\alpha_\nu = \frac{h\nu}{c}\left[n_\ell B_{\ell u} - n_u B_{u\ell}\right]\phi_\nu$$

energy density convention

absorption

stimulated emission is "negative absorption"

Let's convert all Einstein Bs → As ← something "easy" to calculate/lookup

$$\alpha_\nu = \frac{h\nu}{c}\phi_\nu\left[n_\ell \frac{g_u}{g_\ell}\frac{c^3}{8\pi h\nu^3}A_{u\ell} - n_u \frac{c^3}{8\pi h\nu^3}A_{u\ell}\right]$$

$$= \frac{\cancel{h\nu}}{\cancel{c}}\phi_\nu \frac{c^{\cancel{3}\,2}}{8\pi \cancel{h}\nu^{\cancel{3}\,2}}A_{u\ell}\left[n_\ell \frac{g_u}{g_\ell} - n_u\right]$$

Let's pull a factor of n_u out of term in [ ]s:

$$\alpha_\nu = \frac{c^2}{8\pi\nu^2}A_{u\ell}\phi_\nu \, n_u\left[\frac{n_\ell}{n_u}\frac{g_u}{g_\ell} - 1\right]$$

Boltzmann: $\frac{n_u}{n_\ell} = \frac{g_u}{g_\ell}e^{-h\nu/kT_{ex}}$ ⟹ $\frac{n_\ell}{n_u}\cdot\frac{g_u}{g_\ell} = e^{+h\nu/kT_{ex}}$

$$\alpha_\nu = \frac{c^2}{8\pi\nu^2}A_{u\ell}\left[e^{h\nu/kT_{ex}} - 1\right]n_u\,\phi_\nu$$

$$\tau_\nu = \int \alpha_\nu \, ds = \frac{c^2}{8\pi\nu^2}A_{u\ell}\left[e^{h\nu/kT_{ex}} - 1\right]N_u\,\phi_\nu$$

True If Tex const along line of sight.

Optical Depth ~ Column Density (in upper state)