# Absorption & Line Profiles



Let's start with the 1D solution to radiative transfer with constant Source function ($B_\nu(T_{ex})$) along the line of sight:

$$I_\nu = I_\nu(0)\, e^{-\tau_\nu} + B_\nu(T_{ex})\left(1 - e^{-\tau_\nu}\right)$$

$erg \cdot s^{-1} \cdot cm^{-2} \cdot Ster^{-1} \cdot Hz^{-1}$

Real observations occur with a solid ∢ of $\Omega$, so we actually observe the flux density:

$$F_\nu = I_\nu \Omega = F_\nu(0)\, e^{-\tau_\nu} + \underbrace{B_\nu(T_{ex})\,\Omega\left(1 - e^{-\tau_\nu}\right)}_{\to\, 0 \text{ for pure absorption}}$$

$erg \cdot s^{-1} \cdot cm^{-2} \cdot Hz^{-1}$

Since $\tau_\nu \sim \phi_\nu$, the above equation describes an absorption spectrum as a function of $\nu$ or $v$.

The line profile function is <u>typically</u> made up of 2 underlying functions:

## Gaussian Profile
**due to Doppler Motions:**

$$\phi_\nu = \underbrace{\frac{1}{\sqrt{2\pi}} \cdot \frac{1}{\sigma_v}}_{} \cdot \frac{c}{\nu_{ul}}\, e^{-\frac{v^2}{2\sigma_v^2}}$$

Normalized such that $\int \phi_\nu = 1$

convert from $\nu \to v$

$d\nu = dv \cdot \frac{\nu_{ul}}{c}$

Doppler

expected from Maxwell-Boltzmann Distribution

$\times$

## Lorentzian Profile
**due to Intrinsic linewidth (Heisenberg Uncertainty)**

$$\phi_\nu^{intr} = \frac{4\Gamma}{16\pi^2(\nu - \nu_{ul})^2 + \Gamma^2}$$

$$\Gamma = \sum_{E_j < E_u} A_{uj} + \sum_{E_j < E_u} A_{ej}$$

when spontaneous decay dominates

Typically $\phi_\nu^{intr}$ has much smaller linewidth than Gaussian profile but, especially for high $\tau_\nu$ (or Pressure broadening in atmosphere) the wings of $\phi_\nu^{intr}$ dominates over the Gaussian.

For a transition $u \to l$ Heisenberg Uncertainty:

$$\Delta E \, \Delta t \gtrsim \hbar = \frac{h}{2\pi}$$

$$\Delta t \sim \frac{1}{A_{ul}}$$ ← Note this is true for ground state lines, otherwise have to $\Sigma$ over all possible "As" out of u & l state.

$$\Delta E = \cancel{h} \, \Delta\nu^{intr} \sim \frac{\cancel{h}}{2\pi} A_{ul}$$

$$\Delta V^{intr} = \Delta\nu^{intr} \cdot \frac{c}{\nu_{ul}}$$

Let's compare to thermal broadening at $T_K$:

$$\sigma_v = \left(\frac{k T_K}{m}\right)^{1/2} \Rightarrow \Delta V^{therm}_{fwhm} = \left(\frac{8 \ln k T_K}{m}\right)^{1/2}$$

$$\sim 0.215 \left(\frac{T/1\,K}{M/1\,amu}\right)^{1/2} \, km/s$$

For Hydrogen Ly $\alpha$ $\Delta V^{intr}_{fwhm} \sim 0.0121$ km/s

Ly$\alpha$ observed in HII regions $T_K \sim 10^4$ K

$$\Rightarrow \Delta V^{therm}_{fwhm} \sim 21.5 \text{ km/s}$$

$$\Rightarrow \Delta V^{therm} \gg \Delta V^{intr}.$$