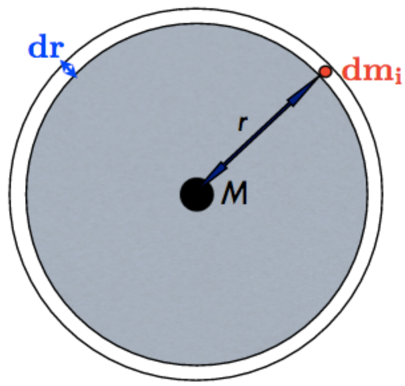
AST 250 – Spring 2018

Homework Due: Wednesday February 7

14. In this problem, you will derive the gravitational potential energy, U_g, of a uniform density sphere (ρ = constant). Consider the geometry of a test mass, dm_i, within a thin spherical shell with differential radius dr:



The test mass has a gravitational potential energy of:

$$dU_{g,i} = -G\frac{M_r\, dm_i}{r}$$

where M_r is the total mass interior to radius r. The gravitational potential energy of the shell is then found by substituting
dm = volume of shell * density = $4\pi r^2 dr * \rho$ to obtain

$$dU_g = -G\frac{M_r\, 4\pi r^2 \rho}{r}\, dr$$

(a) To calculate the total gravitational potential energy of a sphere U_g, we need to integrate dU_g over all mass shells from the center to radius R. Write down this integral and pull everything that doesn't depend on r outside the integral. NOTE: ***capital*** R is the total radius of the sphere.
(b) How is M_r related to r and ρ? Assume ρ is constant. Substitute for M_r and evaluate the integral. Your answer should only contain numbers, G, and ρ and R as variables.
(c) Now substitute for ρ to convert you answer to only have numbers, G, and M (total mass) and R as variables.
(d) As the Sun contracted to its present radius, the virial theorem states that half of the total gravitational potential energy ($U_g/2$) was converted into kinetic energy and half radiated away. At the present luminosity of the Sun, how long would it take the Sun to radiate that energy away (answer in years)?   Can this account for the 4.6 Gyr lifetime of the Sun?