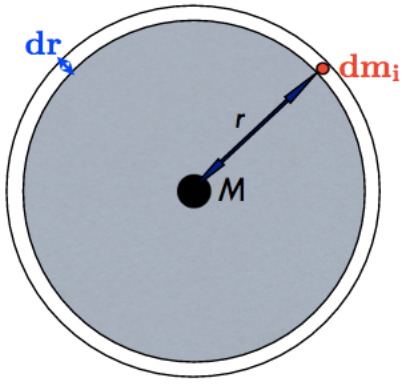
AST 250 – Fall 2017
Homework Due: Friday September 15

11. In this problem, you will derive the gravitational potential energy, U_g, of a uniform density sphere (ρ = constant). Consider the geometry of a test mass, dm_i, within a thin spherical shell with differential radius dr:



The test mass has a gravitational potential energy of:

$$dU_{g,i} = -G\frac{M_r\, dm_i}{r}$$

where M_r is the total mass interior to radius r. The gravitational potential energy of the shell is then found by substituting dm = volume of shell * density = $4\pi r^2 dr * \rho$ to obtain

$$dU_g = -G\frac{M_r\, 4\pi r^2 \rho}{r}\, dr$$

(a) To calculate the total gravitational potential energy of a sphere U_g, we need to integrate dU_g over all mass shells from the center to radius R. Write down this integral and pull everything that doesn't depend on r outside the integral. NOTE: ***capital*** R is the total radius of the sphere.
(b) How is M_r related to r and ρ? Assume ρ is constant and does not depend on r. Substitute for M_r and evaluate the integral. Your answer should only contain numbers, G, and ρ and R as variables.
(c) Now substitute for ρ to convert you answer to only have numbers, G, and M (total mass) and R as variables.
(d) As the Sun contracted to its present radius, the virial theorem states that half of the total gravitational potential energy ($U_g/2$) was converted into kinetic energy. At the present luminosity of the Sun, how long would it take the Sun to radiate that energy away (answer in years)?