Blackbody Radiation

ASTR 250

A blackbody is a perfect absorber and emitter of light. It is a "thermal continuum".

In your Junior level Quantum Mechanics class, you will derive the Planck function. The flux (monochromatic) that emerges from the surface of a blackbody is given by:

Monochromatic Flux emerging from surface of blackbody $= \pi \cdot B_{\nu}(T) = \pi \cdot \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}$

$erg \cdot s^{-1} \cdot cm^{-2} \cdot Hz^{-1}$

monochromatic

$$\pi B_{\lambda}(T) = \pi \cdot \frac{c}{\lambda^2} B_{\nu}(T) \qquad \left(\text{because } |d\nu| = \frac{c}{\lambda^2} |d\lambda| \right)$$

$$\pi B_{\lambda}(T) = \pi \cdot \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} \qquad erg \cdot s^{-1} \cdot cm^{-2} \cdot cm^{-1}$$

(Sometimes also μm^{-1}, nm^{-1}, Å^{-1} etc.)

If we integrate over all ν or λ:

Total Flux of Blackbody $= F = \pi \cdot \int_0^{\infty} B_{\nu}(T)\, d\nu = \sigma T^4$

Stephan-Boltzmann Constant

$$\sigma = 5.67 \times 10^{-5} \; erg \cdot s^{-1} \cdot cm^{-2} \cdot K^{-4}$$

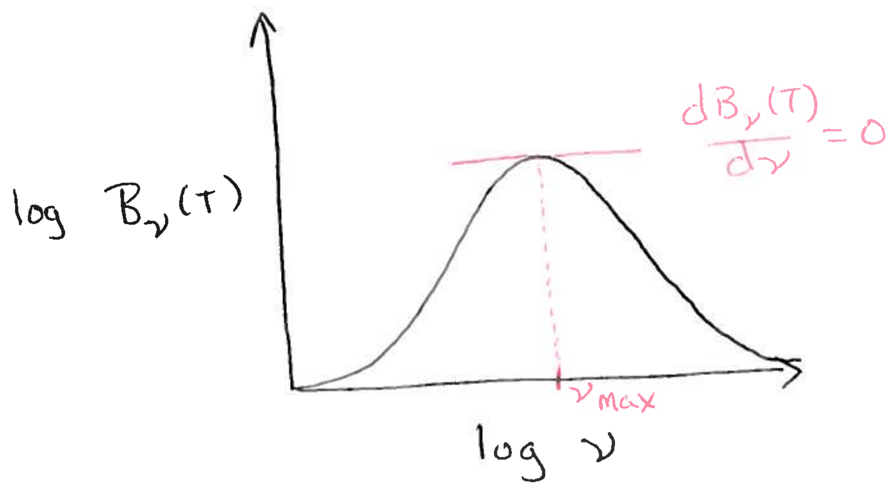
$$= \frac{2 k^4 \pi^5}{c^2 h^3 \cdot 15}$$

The luminosity of a star is then:

$$L = 4\pi R^2 F = \boxed{4\pi R^2 \sigma T^4} \quad erg \cdot s^{-1}$$

NOTE: $L \sim R^2 T^4 \Rightarrow$ Bigger stars more luminous
Hotter stars more luminous

At which ν or λ does a blackbody shine the brightest? Let's differential the blackbody curve:



Wein's Law:

$$\frac{dB_\nu(T)}{d\nu} = 0 \Rightarrow \frac{\nu_{max}}{T} = 5.879 \times 10^{10} \ Hz \cdot K^{-1}$$

$$\frac{dB_\lambda(T)}{d\lambda} = 0 \Rightarrow \lambda_{max} \cdot T = 0.2898 \ cm \cdot K$$

NOTE $\nu_{max} \cdot \lambda_{max} \neq C$ because $d\nu \neq d\lambda$!

$B_\nu(T)$ and $B_\lambda(T)$ are different curves.

Example:

Cosmic Microwave Background is a blackbody to very high accuracy.

$$T_{cmb} = 2.725 \text{ K}$$

$$\lambda_{max} = \frac{0.2898 \text{ cm} \cdot \cancel{\text{K}}}{2.725 \ \cancel{\text{K}}} = 0.106 \text{ cm} = 1.06 \text{ mm}$$

$$\nu_{max} = 5.879 \times 10^{10} \text{ Hz} \cdot \cancel{\text{K}^{-1}} \cdot 2.725 \ \cancel{\text{K}} = 1.60 \times 10^{11} \text{ Hz}$$
$$= 160 \text{ GHz}$$

Note if we calculate $\lambda = \frac{c}{\nu_{max}} = 1.9 \text{ mm} \neq \lambda_{max}$!

Remember $d\nu \neq d\lambda$

$B_\nu(T)$ and $B_\lambda(T)$ are different functions!

they have different peak λ or ν.