Albedo

Planets and small bodies such as asteroids, comets, etc reflect light at optical λ. They are not perfect blackbody absorbers. The fraction of light reflected is called the Albedo (denoted A)

Not to be confused with extinction A_λ!

The flux incident on a planet is:

$$F_{incident} = \frac{L_*}{4\pi d_*^2}$$

← distance between planet and star

$erg \cdot s^{-1} \cdot cm^{-2}$

$$\left(\frac{dE}{dt}\right)_{reflect} \equiv L_{reflect} = A \cdot \pi R_{Pl}^2 \cdot \frac{L_*}{4\pi d_*^2}$$

$erg \cdot s^{-1}$

Albedo × cross-section area × incident flux

$$\left(\frac{dE}{dt}\right)_{absorb} = (1 - A) \cdot \pi R_{Pl}^2 \cdot \frac{L_*}{4\pi d_*^2}$$

↑ reflect + absorb = 100 %

Let's assume that the planet re-radiates the absorbed energy like a blackbody at T_{pl} :

$$L_{planet} = 4\pi R_{pl}^2 \sigma T_{pl}^4$$

In equilibrium, $L_{planet} = \left(\frac{dE}{dt}\right)_{absorb}$ and we can calculate T_{pl} :

$$\cancel{4\pi} \cancel{R_{pl}^2} \sigma T_{pl}^4 = (1-A) \cancel{\pi} \cancel{R_{pl}^2} \frac{L_*}{4\pi d_*^2} \quad L_* = 4\pi R_*^2 \sigma T_*^4$$

$$T_{pl}^4 = \frac{1-A}{4\cancel{\sigma}} \cdot \frac{\cancel{4\pi} R_*^2 \cancel{\sigma} T_*^4}{\cancel{4\pi} d_*^2}$$

$$T_{pl} = T_* \cdot \left(\frac{R_*}{d_*}\right)^{1/2} \cdot \left(\frac{1-A}{4}\right)^{1/4}$$

Note - assumes thermal energy is distributed evenly over $4\pi R_{pl}^2$ surface area (fast rotating or thick atmosphere...)

Plugging in values for Earth / Sun: (using $A \sim 0.37$)

$$T_\oplus \sim 248 \text{ K} \sim -13^\circ\text{F}$$ Below freezing!

The Earth is actually warmer because molecules in the atmosphere (H_2O, CO_2, CH_4, etc.) can absorb IR radiation and effectively act as a blanket = Greenhouse Effect.

Without some greenhouse warming, we would be an ice ball!

The Habitable Zone is calculated by solving for d_* when $T_{pl} = 273 - 373\,K$

(freezing - boiling of H_2O @ STP)

In reality, you need atmospheric models to account for greenhouse effect on T_{pl}.

$$r_{HZ} \sim \frac{\sqrt{(1-A)\,L_*}}{T_{pl}^2} \sim L_*^{1/2}$$