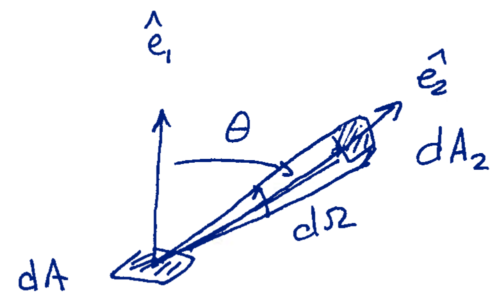
AST 250

Spring 2010

Review: Photometric Concepts

Defn. Monochromatic Specific Intensity



$$I_\nu = \frac{dE}{dt\, dA \cos\theta\, d\Omega\, d\nu} \qquad I_\lambda = \frac{dE}{dt\, dA \cos\theta\, d\Omega\, d\lambda}$$

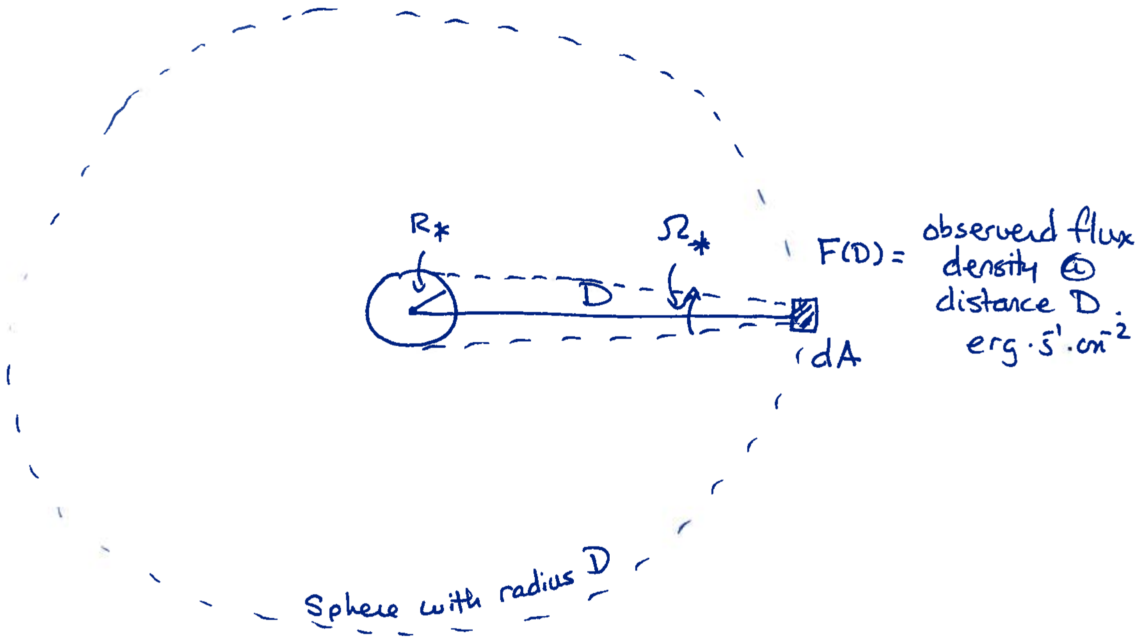
$$d\nu \neq d\lambda \; !$$

Because $\nu = \frac{c}{\lambda} \Rightarrow d\nu = -\frac{c}{\lambda^2} d\lambda$

$\Rightarrow$ Have to specify if per frequency or per wavelength.

Quantity	Definition	units	Differential relation
Monochromatic Specific intensity	$I_\nu = \frac{dE}{dt\, dA\cos\theta\, d\Omega\, d\nu}$	$erg \cdot s^{-1} \cdot cm^{-2} \cdot ster^{-1} \cdot Hz^{-1}$	$\frac{dE}{dt \cos\theta\, dA\, d\Omega\, d\nu}$
Specific intensity	$I = \int_0^\infty I_\nu\, d\nu$	$erg \cdot s^{-1} \cdot cm^{-2} \cdot ster^{-1}$	$\frac{dE}{dt \cos\theta\, dA\, d\Omega}$
Monochromatic Flux Density	$F_\nu = \int_\Omega I_\nu \cos\theta\, d\Omega$	$erg \cdot s^{-1} \cdot cm^{-2} \cdot Hz^{-1}$	$\frac{dE}{dt\, dA\, d\nu}$
Flux Density	$F = \int_\Omega I \cos\theta\, d\Omega$	$erg \cdot s^{-1} \cdot cm^{-2}$	$\frac{dE}{dt\, dA}$
Monochromatic Flux	$\mathcal{F}_\nu = \int_A F_\nu\, dA$	$erg \cdot s^{-1} \cdot Hz^{-1}$	$\frac{dE}{dt\, d\nu}$
Flux	$\mathcal{F} = \int_A F\, dA$	$erg \cdot s^{-1}$	$\frac{dE}{dt}$
Luminosity	$L = \int_{\text{surface area of sphere}} F^+\, dA = 4\pi r^2 F^+$	$erg \cdot s^{-1}$	$\frac{dE}{dt}$

Conservation of Energy



$$L_* = 4\pi R_*^2 F_*^+ \qquad = 4\pi D^2 F_{obs}(D)$$

emergent flux density from surface of star

$$F_{obs}(D) = F_*^+ \left(\frac{R_*^2}{D^2}\right) \quad \sim \frac{1}{D} \quad \text{inverse square law.}$$

Now from the point of view of dA, the star subtends a solid angle

$$\Omega_* = \frac{A_{projected,*}}{D^2} = \frac{\pi R_*^2}{D^2}$$

Let's define the Surface Brightness $\equiv$

$$B = \frac{F_{obs}(D)}{\Omega_*} = \frac{F_*^+ \cdot \frac{R_*^2}{D^2}}{\frac{\pi R_*^2}{D^2}} = \frac{F_*^+}{\pi}$$

Remember that $F_*^+ = \pi \cdot I$ ("astrophysical" flux density)

so $B = I$ surface brightness is really just specific intensity

AND $I \sim \text{CONST} \sim \frac{F_*^+}{\pi}$ is Distance independent!

Magnitudes

Hipparcos 1st defined 6 Classes ~ 2nd century B.C.

Each class was ~ factor of 2 in brightness
1st class was brightest and 6th class dimmest.

Norman Pogson (1856) creates modern definition of magnitudes

$$\text{ratio of brightness of 1 magnitude} = \frac{F_m}{F_{m+1}} = \sqrt[5]{100} \simeq 2.512$$

why the $\sqrt[5]{100}$?! Because this means a difference of 5 magnitudes is a factor of 100 in brightness.

$$\text{apparent magnitude} = m \equiv -2.5 \log_{10} \frac{F}{F_{m=0}}$$

F ← flux density $erg \cdot s^{-1} \cdot cm^{-2}$

$F_{m=0}$ ← normalized to flux density of magnitude 0.

m: lower case

negative sign means smaller m is brighter!

Difference in magnitudes between two stars:

$$m_1 - m_2 = -2.5 \log_{10} \frac{F_1}{F_{m=0}} - \left(-2.5 \log_{10} \frac{F_2}{F_{m=0}}\right) = -2.5 \log_{10} \frac{F_1}{F_2}$$

Modern Magnitudes defined in a filter system:

Johnson & Morgan Filters	U	B	V	R	I
center λ:	367 nm	436 nm	545 nm	638 nm	797 nm

Created in 1950s

$m_V = 0$ normalized to an A0 star (e.g. used to be Vega but modern corrections show Vega has $m_V = 0.03$ mag)

Magnitudes
CON'T.

Define

Absolute Magnitude = apparent magnitude of a star @ 10 pc = M_v

Capital M — subscript indicates filter

$$m - M = -2.5 \log_{10} \frac{F(D)}{F(10pc)}$$

Since $F(D) \sim \frac{1}{D^2}$

$$m - M = -2.5 \log_{10} \left(\frac{10}{D}\right)^2 = \underbrace{+5 \log_{10} \left(\frac{D_{(pc)}}{10pc}\right)}_{\text{This term is called the "Distance Modulus"}}$$

For reference M_v for the sun is $M_{v,\odot} = +4.83$ mag

Colors

Color = Difference between 2 magnitudes   e.g. B-V   U-B   etc.

B-V: shorthand for $m_B - m_V$

In the Johnson-Morgan System a star of spectral type A∅ (like Vega) has colors defined B-V = U-B = 0.0

Example: The Sun $m_v = -26.8$ mag   B-V = 0.62 mag
$M_v = +4.83$ mag   U-B = 0.10 mag

<u>Problem:</u> The limit of the dark adapted eye is $m_v \sim 6$ mag. How far away from the Earth could a star like the sun still be visible?

$$M_{v,\odot} = +4.83 \text{ mag}$$

$$m_v = +6.0 \text{ mag}$$

← want to know D of * that would appear w/ $m_v = +6.0$ mag

$$m_v - M_v = +5 \log_{10}\left(\frac{D_{pc}}{10pc}\right)$$

Solve for D:

$$\log_{10}\left(\frac{D_{pc}}{10pc}\right) = \frac{1}{5}(m_v - M_v)$$

$$\frac{D_{pc}}{10\,pc} = 10^{0.2(m_v - M_v)}$$

$$\frac{D}{10pc} = 10^{0.2(6.0 - 4.83)} \Rightarrow \frac{D}{10pc} = 1.71$$

$$D = 17.1 \text{ pc} \times \frac{3.26 \text{ ly}}{\cancel{3.26}\ 1 \text{ pc}} = 55.9 \text{ ly}$$

Anothe example:

Binary Stars with magnitudes m_1 & m_2

If the two stars are unresolved by the eye, what is the total magnitude?

$$m_{tot} \neq m_1 + m_2 \quad !!$$

$$m_1 = -2.5 \log_{10} \frac{F_1}{F_{m=0}} \qquad m_2 = -2.5 \log_{10} \frac{F_2}{F_{m=0}}$$

$$F_1 = F_{m=0} \cdot 10^{-0.4 m_1} \qquad F_2 = F_{m=0} \cdot 10^{-0.4 m_2}$$

The total observed flux density is

$$F_{tot} = F_1 + F_2 = F_{m=0} \left(10^{-0.4 m_1} + 10^{-0.4 m_2} \right)$$

So the total magnitude is:

$$m_{tot} = -2.5 \log_{10} \frac{\cancel{F_{m=0}} \left(10^{-0.4 m_1} + 10^{-0.4 m_2} \right)}{\cancel{F_{m=0}}}$$