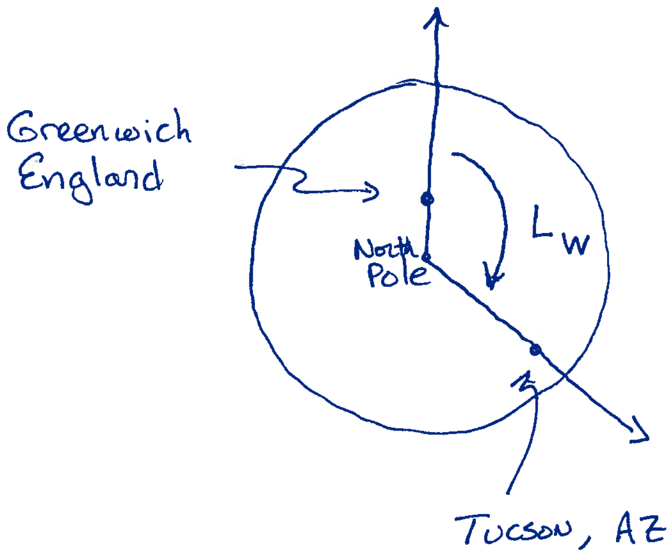
The LST depends on your position on the Earth!

Looking down on the Earth:



The difference between directions in space is just L_W.

So $$LST = GST + L_W$$

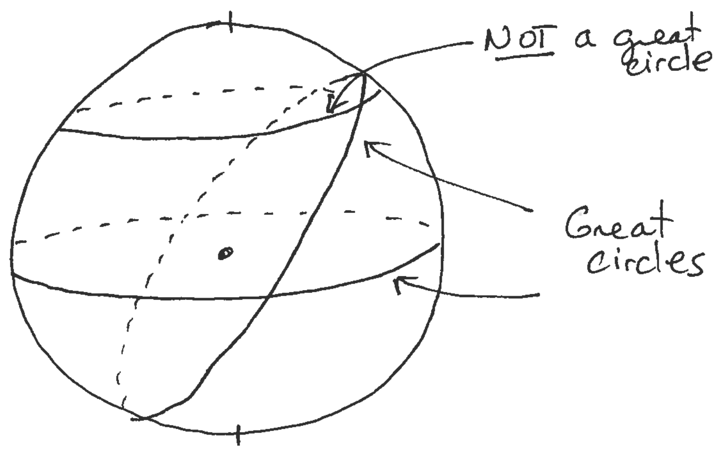
GST: Local Sidereal Time in Greenwich, England.

L_W: Longitude of the observer

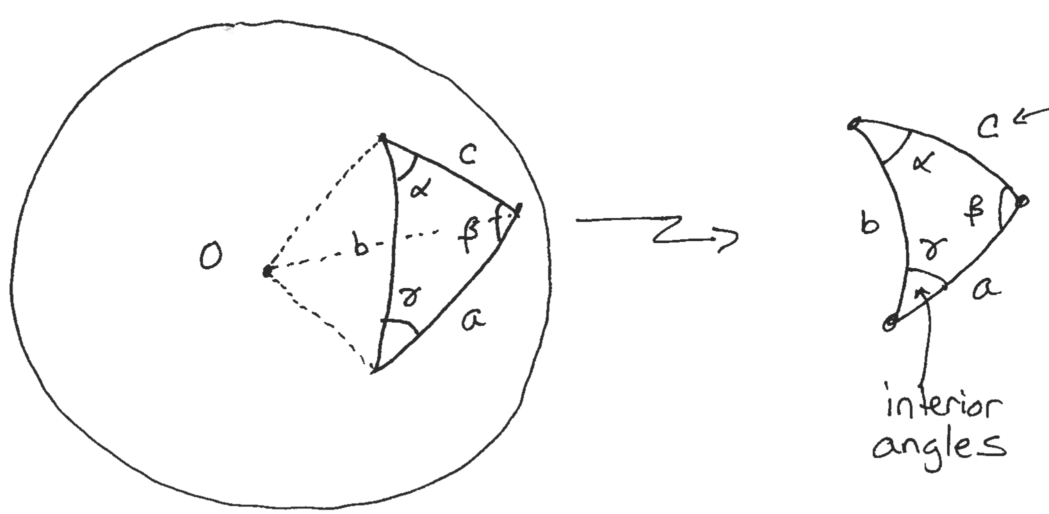
GST is tabulated in the Astronomical Almanac Section B.

Spherical Trigonometry

Defn. : Great Circle ≡ a circle on the surface of a sphere whose center is the center of the sphere



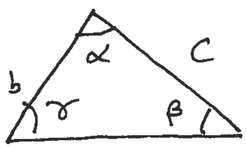
Defn : Spherical Triangle ≡ a triangle on the surface of a sphere whose sides are made of 3 great circles



notice that the "sides" are really angles subtended from the center of the sphere.

Euclidean Geometry (Flat Space)

Law of Cosines



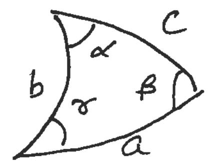
$$a^2 = b^2 + c^2 - 2ab\cos\alpha$$

Law of Sines

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

Spherical Geometry ("Non-Euclidean")

Law of Cosines



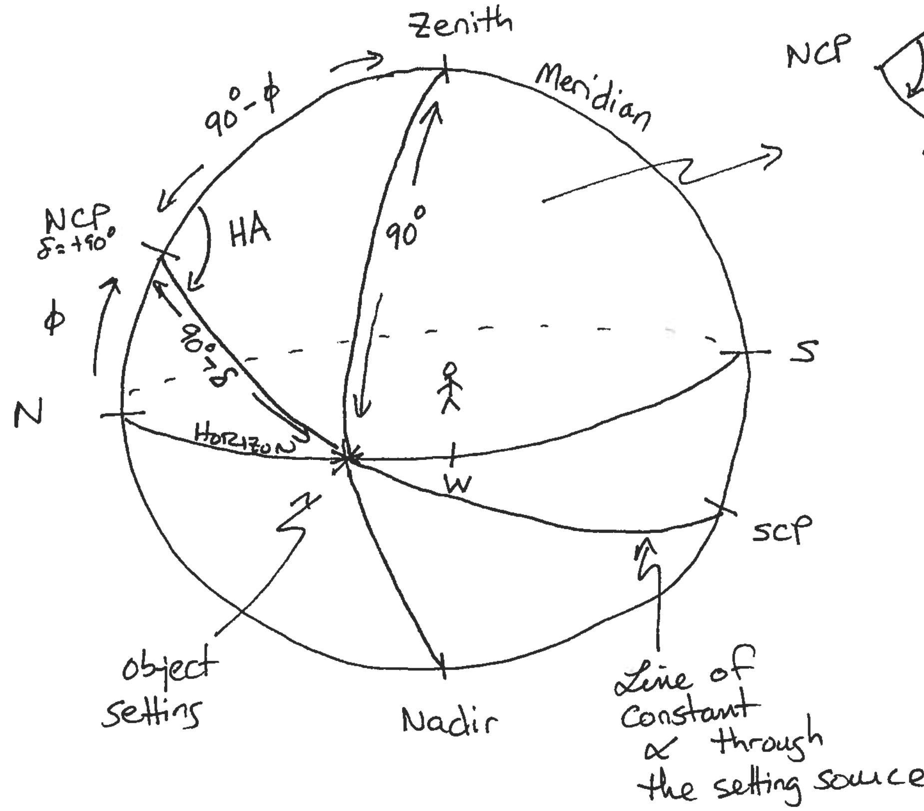
$$\cos a = \cos b \cos c + \sin b \sin c \cos\alpha$$

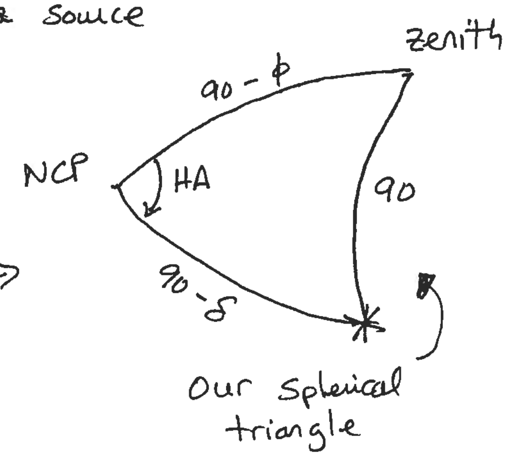
Law of Sines

$$\frac{\sin\alpha}{\sin a} = \frac{\sin\beta}{\sin b} = \frac{\sin\gamma}{\sin c}$$

see online notes for a derivation of these equations or Fundamental Astronomy §2.1

Example: What is the HA of a source that is setting?





Using the law of cosines:

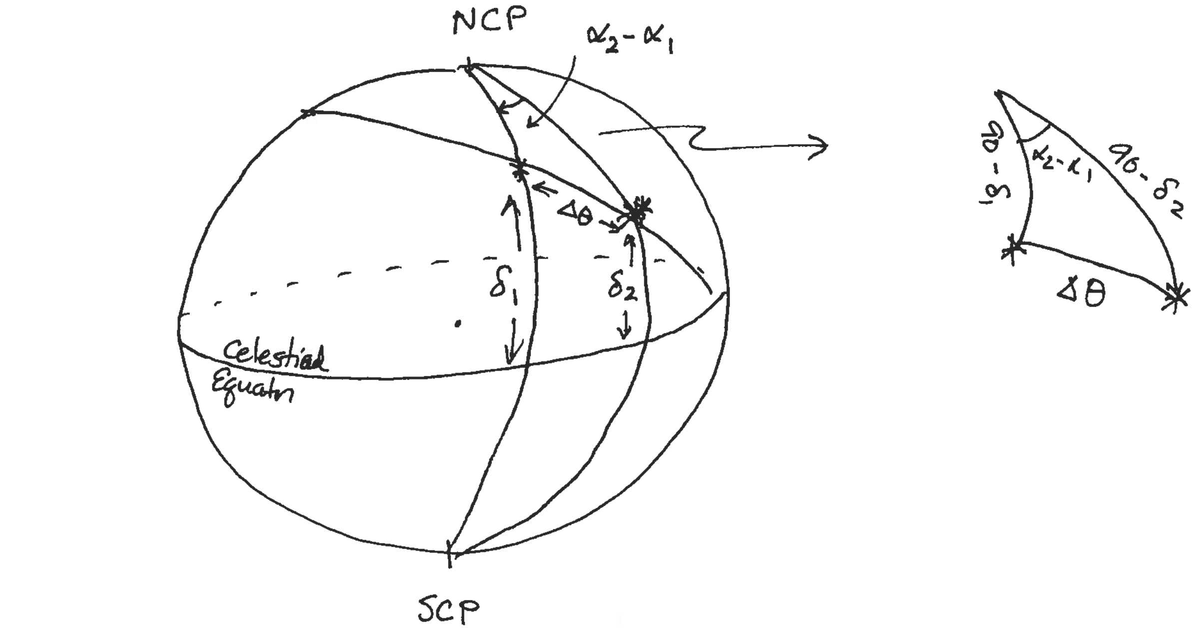
$$\cos 90^\circ = \cos(90-\phi)\cos(90-\delta) + \sin(90-\phi)\sin(90-\delta)\cos HA$$

$$0 = \sin\phi\,\sin\delta + \cos\phi\,\cos\delta\,\cos HA$$

$$\cos HA = -\frac{\sin\phi\,\sin\delta}{\cos\phi\,\cos\delta} = -\tan\phi\,\tan\delta$$

$$HA = \cos^{-1}\left[-\tan\phi\,\tan\delta\right]$$

Another Example: What is the angular separation between two stars with coordinates (α_1, δ_1) (α_2, δ_2) ?



USING Law of Cosines:

$$\cos\Delta\theta = \cos(90-\delta_1)\cos(90-\delta_2) + \sin(90-\delta_1)\sin(90-\delta_2)\cos(\alpha_1-\alpha_2)$$

$$\cos\Delta\theta = \sin\delta_1 \sin\delta_2 + \cos\delta_1 \cos\delta_2 \cos(\alpha_1-\alpha_2)$$

Let's do a numerical example: $(\alpha_1, \delta_1) = (10^h, +80^\circ)$

$(\alpha_2, \delta_2) = (11^h, +70^\circ)$

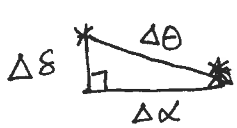
$$\alpha_2 - \alpha_1 = 11^h - 10^h = 1^h \times \frac{360^\circ}{24^h} = 15^\circ$$

So we have

$$\cos\Delta\theta = \sin 80^\circ \sin 70^\circ + \cos 80^\circ \cos 70^\circ \cos 15^\circ$$

$$\Rightarrow \boxed{\Delta\theta = 10.^\circ6}$$ Correct answer.

For comparison, what if we used the Pythagorean theorem for plane triangles:

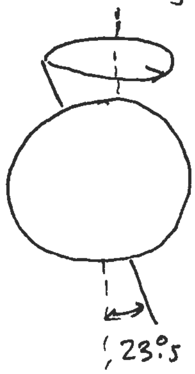


$$\Rightarrow \Delta\theta^2 = (15^\circ)^2 + (10^\circ)^2 \Rightarrow \Delta\theta = 18^\circ$$ incorrect!

$\leftarrow$ Notice that not taking the curvature on the sphere causes an error of $7.^\circ4$ for even these closely spaced stars!

Epochs

Unfortunately, the Equitorial Coordinate system is not fixed.



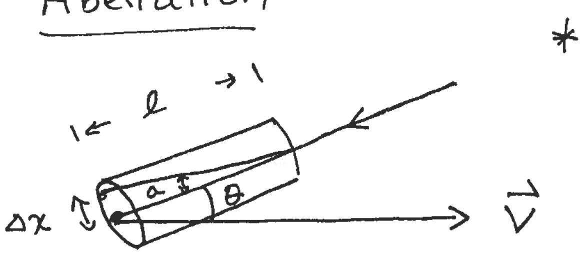
The Earth precesses on its axis every ~ 26,000 years ⇒ Direction of NCP slowly changes.

⇒ when you give coordinates (α, δ), you must also give the epoch for when those coordinates are true!

e.g. $5^h\, 30^m\, 40.^s2 \quad +17^\circ\, 8'\, 33''$ J2000.0 — the epoch.

Precession accounts for motion of the vernal equinox along the ecliptic (plane of the Earth's orbit) of ~ 50"/year.

Aberration



Imagine you have a telescope of length l moving at velocity $\vec{V}$ with respect to a star.

In the time $t = \frac{l}{c}$ it takes the light to traverse the telescope, the light beam will be displaced by Δx.

$$\Delta x = \left(\text{velocity component} \perp \text{to light beam}\right) \cdot t$$

$$= V \cdot \sin\theta \cdot t$$

$$= V \cdot \sin\theta \cdot \frac{l}{c}$$

So, $$a = \frac{\Delta x}{l} = \frac{V}{c} \cdot \sin\theta$$

The motion of the Earth around the Sun ($V_\oplus \sim 30$ km/s) accounts for up to 21" in aberration angle!