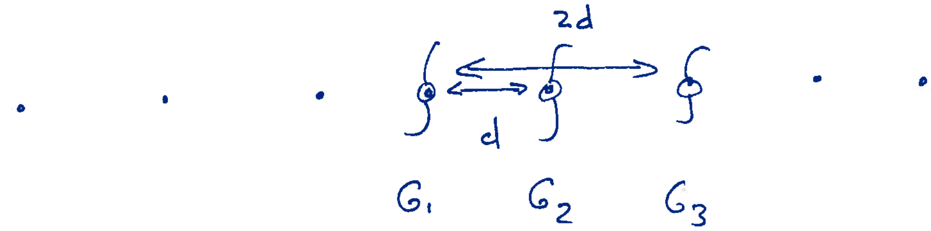
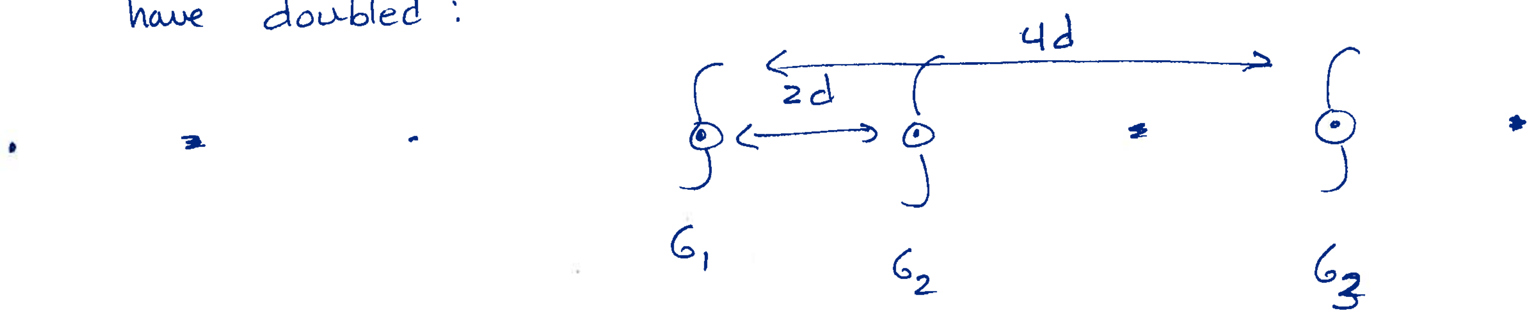
Expansion of Space-time

Imagine you have a set of points. ("co-moving coordinates")



with three galaxies G_1, G_2, G_3. From the point-of-view of G_1, G_2 & G_3 are d and 2d away.

If this 1D space-time expands at a constant rate, at some time Δt later, the distance between points will have doubled:



The velocity of G_2: $V = \dfrac{\Delta d}{\Delta t} = \dfrac{2d - d}{\Delta t} = \dfrac{d}{\Delta t}$

The velocity of G_3: $V = \dfrac{\Delta d}{\Delta t} = \dfrac{4d - 2d}{\Delta t} = \dfrac{2d}{\Delta t}$

$\Rightarrow \quad V \sim d$ The velocity of expansion is linearly proportional to the distance away from G_1

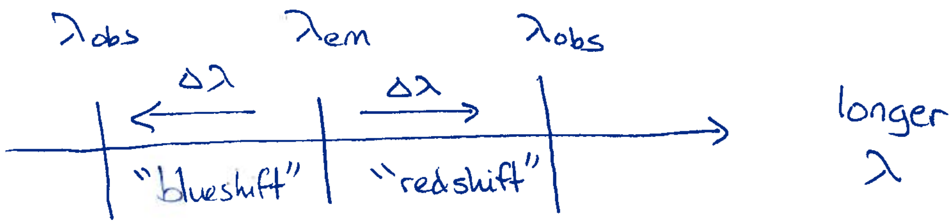
This is a "Hubble Law".

Also note that our choice of G_1 was arbitrary $\Rightarrow$ there is no "center" in this expansion.

How we measure this velocity (or distance) is from the shift in λ of observed spectral lines in the Galaxy.

λ_{obs} = observed wavelength

λ_{em} = emitted wavelength in rest frame of the object



Definition of "redshift" z:

$$\boxed{z \equiv \frac{\Delta\lambda}{\lambda_{em}} = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}}}$$

$$z = \frac{\lambda_{obs}}{\lambda_{em}} - 1$$

$$\Rightarrow 1 + z = \frac{\lambda_{obs}}{\lambda_{em}}.$$

Also, since $\nu \cdot \lambda = c$

$$1 + z = \frac{\nu_{em}}{\nu_{obs}}$$

Example : Ly α $\lambda_{rest} = 121.567$ nm.

For a galaxy at $z = 1$:

$$\lambda_{obs} = (1+z) \cdot \lambda_{em}$$
$$= (1+1) \cdot \lambda_{em}$$
$$= 2\lambda_{em}$$

If we can identify a spectral line in a source, we can determine its redshift, z.

In General, the observed redshift may be comprised of :

1.) Doppler Shift or "Peculiar Velocity" $\frac{\lambda_{obs}}{\lambda_{em}} = 1 + z = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}$

special relativity

2.) Expansion of spacetime e.g. $v = H_0 d$ Hubble Law

3.) Gravitational redshift : $\frac{\nu_{obs}}{\nu_{em}} = \sqrt{1 - \frac{2GM}{c^2 d}}$ (non-rotating metric)

Doppler Shifts

How does $\frac{\lambda_{obs}}{\lambda_{em}} = 1 + z = \sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}$ behave for $v \ll c$?

Taylor Expansions:

$$\left(1+\frac{v}{c}\right)^{1/2} \simeq 1 + \frac{1}{2}\frac{v}{c} + O\left(\frac{v^2}{c^2}\right) \cdots$$

$$\left(1-\frac{v}{c}\right)^{-1/2} \simeq 1 + \frac{1}{2}\frac{v}{c} + O\left(\frac{v^2}{c^2}\right) \cdots$$

So, $$1 + z = \left(1+\frac{v}{c}\right)^{1/2} \cdot \left(1-\frac{v}{c}\right)^{-1/2} \simeq \left(1 + \frac{1}{2}\frac{v}{c}\right)\left(1 + \frac{1}{2}\frac{v}{c}\right)$$

$$\simeq 1 + \frac{v}{c} + \cancel{\frac{1}{4}\frac{v^2}{c^2}} \rightarrow \text{small } O\left(\frac{v^2}{c^2}\right)$$

$\Rightarrow \quad v = cz \quad$ non-relativistic

also $\Rightarrow$ since $z = \frac{\Delta\lambda}{\lambda_{em}} \Rightarrow v = c \cdot \frac{\Delta\lambda}{\lambda_{em}}$

The doppler shift velocity cannot exceed c.

If $\frac{v}{c} \gtrsim 0.5$ should definitely use relativistic formula.

This doppler redshift is <u>NOT</u> the same as expansion of the universe !!!

Hubble Law

1929 Edwin Hubble determined $V = H_0 \cdot d$

$\uparrow$

Hubble's constant

H_0 ← means measured at present epoch

Current best estimate from WMAP + SNe + BAO (~2009):

$$H_0 = 70.5 \pm 1.3 \ km/s/Mpc$$

$\uparrow$

Notice units.

example: Galaxy with $v = 1000$ km/s

$$d = \frac{V}{H_0} = \frac{1000 \ \cancel{km/s}}{70.5 \ \cancel{km/s}/Mpc} = 14.2 \ Mpc$$

example: At what distance does a galaxy have $\frac{V}{c} = 0.1$ (10% speed of light)?

$$d = \frac{3 \times 10^4 \ km/s}{70.5 \ km/s/Mpc} = 425 \ Mpc$$

For $v \lesssim (0.3 - 0.4)\,c$ then $v = cz$, so $cz = H_0 d$.

For $z > 0.4$, you should start using a cosmological model to calc v.