ASTR 250 HMWK 8 SPRING 2010

(1.)

$$V = c \cdot z = 3\times10^{5}\ \mathrm{km/s} \cdot 0.0736 = 2.2\times10^{4}\ \mathrm{km/s}$$

$$d = \frac{V}{H_0} = \frac{2.2\times10^{4}\ \mathrm{km/s}}{71.5\ \mathrm{km/s/Mpc}} = 309\ \mathrm{Mpc}$$

$$m_V - M_V = 5\log_{10}\frac{d}{10\,\mathrm{pc}}$$

$$M_V = +17.1 - 5\log_{10}\frac{309\times10^{6}\,\mathrm{pc}}{10\,\mathrm{pc}} = +17.1 - 37.4$$

$$= -20.3\ \mathrm{mag}$$

$$M_V - M_V^{\mathrm{typical}} = -2.5\log_{10}\frac{L}{L^{\mathrm{typical}}}$$

$$\frac{L}{L^{\mathrm{typical}}} = 10^{-0.4\,(M_V - M_V^{\mathrm{typical}})} = 10^{-0.4\,(-20.3+19.3)}$$

$$= 2.6$$

so this supernova was $2.6\times$ overluminous

(2.)

$$1 + q = -\frac{1}{H^2}\frac{dH}{dt} = -\frac{1}{H^2}\dot{H} \quad \text{where } \dot{H} = \frac{dH}{dt}$$

but $H = \frac{\dot{a}}{a}$ so $\dot{H} = \frac{dH}{dt} = \frac{d}{dt}\left(\frac{\dot{a}}{a}\right) = \frac{\ddot{a}a - \dot{a}\dot{a}}{a^2} = \frac{\ddot{a}a}{a^2} - \frac{\dot{a}^2}{a^2}$

substituting for $\frac{dH}{dt}$: $1 + q = -\frac{1}{H^2}\left(\frac{\ddot{a}a}{a^2} - H^2\right)$

$$\cancel{1} + q = -\frac{\ddot{a}a}{a^2H^2} \cancel{+1}$$

$$q = -\frac{\ddot{a}a}{\cancel{a^2}}\cdot\frac{\cancel{a^2}}{\dot{a}^2} = -\frac{\ddot{a}a}{\dot{a}^2}$$

Q.E.D.

(3.) If $q = 0 \Rightarrow -\int \frac{1}{H^2}\,dH = \int dt \Rightarrow t = \frac{1}{H_0}$

$$t = \frac{1}{H_0} = \frac{1}{71.5\,(\text{km/s})/\text{Mpc} \times \dfrac{1\,\text{Mpc}}{3.09\times10^{18}\,\frac{\cancel{\text{cm}}}{\cancel{\text{pc}}} \times 1.0\times10^{6}\,\frac{\cancel{\text{pc}}}{\text{Mpc}} \times \frac{1\,\text{km}}{10^5\,\cancel{\text{cm}}}}}$$

$$t = 4.32\times10^{17}\,\text{s} \times \frac{1\,\text{yr}}{3.1\times10^{7}\,\text{s}} \simeq 13.9 \text{ billion years}$$

(4.) Wein's Law: $\lambda_{max}\cdot T = K$ = constant = 0.2898 cm·K

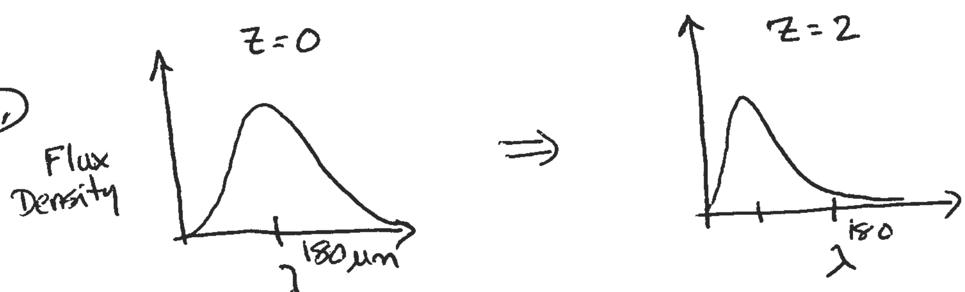
This is true at all redshifts for a blackbody.

$$T = \frac{K}{\lambda_{max}}$$

From definition of redshift: $1+z = \dfrac{\lambda_{obs}}{\lambda_{em}} = \dfrac{\left(\frac{K}{T_{obs}}\right)}{\left(\frac{K}{T_{em}}\right)} = \dfrac{T_{em}}{T_{obs}}$.

Q.E.D.

(5.)



$$T_{em} = T_{obs}(1+z) = \frac{K}{\lambda_{max}^{obs}}(1+z) = \frac{0.2898\,\text{cm}\cdot\text{K}}{180\times10^{-4}\,\text{cm}}(1+2)$$

$$\boxed{T_{em} \simeq 48\text{ K}}$$

If we didn't account for redshift, we would have thought the galaxy was only 16 K.