HMRK #7 Solutions

(#1.)

$$m_V^{obs} - M_V = 5\log_{10}\left(\frac{d^{true}}{10}\right) + A_V$$

$m_V^{obs} = +10.0$ mag

$M_V = +4.83$ mag

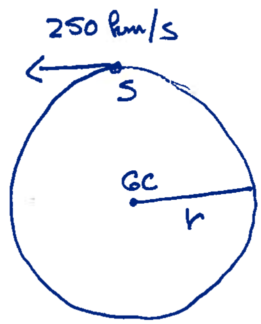
$A_V = 1$ mag

$$\cancel{5}\log_{10}\left(\frac{d}{10}\right) = \frac{(m_V^{obs} - A_V - M_V)}{5}$$

$$\frac{d_{pc}}{10} = 10^{\frac{(m_V^{obs} - A_V - M_V)}{5}}$$

$$d^{true} = 68.2 \text{ pc}$$

(2.)



$$P = \frac{\text{distance traveled}}{\text{speed}} = \frac{2\pi \cdot r}{V_\odot}$$

$$P = \frac{2\pi \cdot 8.5\text{ kpc} \cdot 1000\frac{\text{pc}}{\text{kpc}} \cdot 3.086\times10^{18}\frac{\text{cm}}{\text{pc}}}{250\text{ km/s} \cdot 10^5\text{ cm/km}}$$

$P = 6.6\times10^{16}$ s $\simeq$ 210 Million years

$\Rightarrow$ Sun has made $\frac{4500}{210} = 21$ orbits over its lifetime.

Two ways to get mass:

$$V_c = \sqrt{\frac{GM}{r}}$$

$$M_{MW}(r) = \frac{r \cdot V_c^2}{G} = \frac{2.62\times10^{22}\text{ cm} \cdot (250\times10^5\text{ cm/s})^2}{6.67\times10^{-8}\text{ cm}^3\cdot\text{s}^{-2}\cdot\text{g}^{-1}}$$

$$= \cancel{1.9\times10^{44}\text{ g}} = \cancel{9.5\times10^{10}}\ M_\odot$$

$$2.4\times10^{44}\text{ g} = 1.2\times10^{11}\ M_\odot$$

$$M_\odot = \frac{a_{AU}^3}{P_{yrs}^2}$$

$$a = r = 2.62\times10^{22}\text{ cm} \cdot \frac{1\text{ AU}}{1.496\times10^{13}\text{ cm}}$$

$$M = \frac{(1.75\times10^9\text{ AU})^3}{(212\times10^6\text{ yrs})^2}$$

$$M = 1.2\times10^{11}\ M_\odot$$

3.) (a) Assuming the Coma Cluster is spherical and virialized, it should not matter which direction we view the cluster – the velocity dispersion of galaxies in the cluster should appear the same.

If V_x = velocity component along our line-of-sight = V_r

Then in 3 dimensions $V_x = V_y = V_z \simeq 700$ km/s

Therefore $|\vec{V}| = \sqrt{V_x^2 + V_y^2 + V_z^2}$ Pythagorean theorem in 3-D.

$$= \sqrt{3} \cdot V_x$$

(b) Virial theorem: $\langle K.E \rangle = -\frac{1}{2} \langle P.E. \rangle$

For a galaxy of mass m_{gal} in the Coma Cluster M_{coma}:

$$\cancel{\tfrac{1}{2}} \cancel{m_{gal}} \langle V^2 \rangle = -\cancel{\tfrac{1}{2}} \cdot -\frac{3}{5} \frac{G M_{coma} \cancel{m_{gal}}}{R_{coma}}$$

substituting from (a): $\cancel{3} \langle V_r^2 \rangle = \frac{\cancel{3}}{5} \frac{G M_{coma}}{R_{coma}}$

$$M_{coma} = \frac{5 R_{coma} \langle V_r^2 \rangle}{G} = \frac{5 \cdot \left(3\,Mpc \cdot 10^6 \frac{pc}{Mpc} \cdot 3.086 \times 10^{18} \frac{cm}{pc}\right) \cdot \left(700\,km/s \cdot 10^5\,cm/s\right)^2}{6.67 \times 10^{-8}\ cm^3 \cdot s^{-2} \cdot g^{-1}}$$

$$M_{coma} = 3.4 \times 10^{48}\,g \cdot \left(\frac{1\,M_\odot}{2 \times 10^{33}\,g}\right) = 1.7 \times 10^{15}\,M_\odot$$

$$\frac{M_{coma}}{L_{coma}} = \frac{1.7 \times 10^{15}\,M_\odot}{3 \times 10^{13}\,L_\odot} \simeq 60\,!$$

$\Rightarrow$ There is a significant fraction of "dark" (non-luminous) matter in the Coma cluster!

(4.) $$\frac{\nu_{em}}{\nu_{obs}} = 1 + z \Rightarrow \nu_{obs} = \frac{\nu_{em}}{1+z}$$

$$\nu_{obs} = \frac{230.538 \text{ GHz}}{(1 + 6.419)} = 31.074 \text{ GHz}$$

This is Ka band for the VLA.

(5.) $$z = \frac{\Delta\lambda}{\lambda} = \frac{6740.3 - 6562.8}{6562.8} = 0.027$$

since z is small we can use

$$V = c \cdot z = (3 \times 10^5 \text{ km/s}) \cdot 0.027 \simeq 8110 \text{ km/s}$$

Hubble Law:

$$d = \frac{V}{H_0} = \frac{8110 \text{ km/s}}{70.5 \text{ km/s/Mpc}} = 115 \text{ Mpc}$$

a little further away than the average distance to galaxies in the Coma Cluster ($d \sim 100$ Mpc)