ASTR 250
HMRK #4

(1.)

$$\bar{\rho}_\odot = \frac{M_\odot}{V_\odot} = \frac{3M_\odot}{4\pi R_\odot^3} = \frac{3}{4\pi}\,\frac{2\times10^{33}\,g}{(6.96\times10^{10}\,cm)^3} \simeq 1.4\ g\cdot cm^{-3}$$

Note that $\rho_{H_2O} = 1.0\,g\cdot cm^{-3}$! so $\bar{\rho}_\odot$ not that different from H_2O !

$$\bar{\rho}_{\alpha\,ORi} = \rho_\odot \cdot \frac{20}{(1000)^3} = 2.8\times10^{-8}\,g\cdot cm^{-3}$$

$$\bar{\rho}_{sirius\,B} = \rho_\odot \cdot \frac{0.98}{(\quad)^3} = 2.3\times10^{6}\,g\cdot cm^{-3}$$

(2.)

$$E_\odot^{lifetime} \simeq L_\odot \cdot t_{life} = 3.9\times10^{33}\,erg\cdot s^{-1} \cdot 5\times10^{9}\,yrs \cdot \frac{3\times10^7\,s}{1\,yr} = 5.8\times10^{50}\,erg$$

Virial theorem

$$E_{grav} = -\frac{1}{2}\mathcal{U} = -\frac{1}{2}\cdot -\frac{3}{5}\frac{GM_\odot^2}{R} = E_\odot^{lifetime}$$

Solving for R:

$$R = \frac{3}{10}\frac{GM_\odot^2}{E_\odot^{life}} = \frac{3}{10}\cdot\frac{(6.67\times10^{-8}\quad)(2\times10^{33}\,g)^2}{5.8\times10^{50}\,erg}$$

$$R = 1.4\times10^{8}\,cm \sim 1400\,km < R_\oplus\ !$$

$$\bar{\rho} = \frac{3M_\odot}{4\pi R^3} = 1.8\times10^{8}\,g\cdot cm^{-3}$$

this is almost 100x denser than a typical $1M_\odot$ white dwarf!!

3.

Flux Density of Neutrinos at the Earth $= F^{+}_{neutrinos} = \dfrac{10^{38}}{4\pi D^2} = \dfrac{10^{38} \text{ neutrinos/s}}{4\pi\,(149.6\times10^{11}\text{cm})^2}$

$$F^{+}_{neutrinos} = 3.5\times10^{10}\ \frac{\text{neutrinos}}{\text{s}\cdot\text{cm}^2}$$

So the Flux of neutrinos passing through your brain is

$$\mathcal{F}_{neutrinos} = F^{+}_{neutrinos}\cdot A_{brain} = 3.5\times10^{10}\ \frac{\text{neutrinos}}{\text{s}\cdot\text{cm}^2}\cdot\frac{\pi d_{brain}^2}{4}$$ ← diameter of brain

<u>estimate:</u> $d_{brain} \simeq \cancel{20}\ 15\text{ cm}$

$$\mathcal{F}_{neutrinos} \approx 3.5\times10^{10}\ \frac{\text{neutrinos}}{\text{s}\cdot\cancel{\text{cm}^2}}\cdot\frac{\pi(15\,\cancel{\text{cm}})^2}{4} = 6.2\times10^{12}\ \frac{\text{neutrinos}}{\text{s}}$$

4.

$\varepsilon_{p\text{-}p} \sim T^4$ so $\dfrac{\varepsilon'_{p\text{-}p}}{\varepsilon_{p\text{-}p}} = \left(\dfrac{T+\Delta T}{T}\right)^4 = \left(\dfrac{1.10}{1.0}\right)^4 \sim 1.46$

So the rate of energy generation would increase by 46% for a 10% increase in T.

(5.)

$$\text{Energy generated by fusion} = E_{\text{fusion}}^{\text{total}} = \underbrace{0.007\,c^2}_{\substack{\text{energy created}\\ \text{per proton}}} \cdot \underset{\substack{\uparrow\\ \text{fraction of}\\ \text{protons burned}\\ \text{by fusion}}}{f} \cdot \underbrace{(0.74\,M_\odot)}_{\substack{\text{total mass of}\\ \text{protons in the}\\ \text{sun}}}$$

$E_{\text{fusion}}^{\text{total}} = E_\odot^{\text{lifetime}} = 5.8 \times 10^{50}$ erg from problem #2.

Solving for f:

$$f = \frac{E_\odot^{\text{life}}}{0.007\,c^2\,(0.74 M_\odot)} = \frac{5.8 \times 10^{50}\ \text{erg}}{0.007\,(3 \times 10^{10}\ \text{cm/s})^2 \cdot (0.74 \cdot 2 \times 10^{33}\ \text{g})} = 0.06$$

$\Rightarrow$ About 6% of the mass of hydrogen has burned.