Nucleosynthesis

Review: Fusion converts $4\,{}^{1}H \rightarrow {}^{4}He$

$$\Delta m = 4m({}^{1}H) - m({}^{4}He) = 0.0291 \text{ amu}$$

$$\frac{\Delta m}{4} = 0.7\%\, m_H$$

$$\Delta E = \Delta m c^2 \sim 27 \text{ MeV}$$

How does this conversion actually occur?
This is what we are going to figure out today

Problem: $(p) \longrightarrow \quad \longleftarrow (p)$

Collision of 2 protons. Protons are + charged. Implies we have to overcome electrostatic repulsion to combine them into a nucleus.

1st - what kind of T can we expect in the Sun?

Let's estimate from the Virial Theorem!

$$\langle K.E. \rangle = -\frac{1}{2}\langle \mathcal{U} \rangle$$

For a "perfect" gas (N = number of gas particles)

$$K.E. = \frac{3}{2} N k T_{vir}$$

$$\mathcal{U} = -\frac{3}{5}\frac{GM^2}{R} \quad \leftarrow \text{ we calculated this last time.}$$

So

$$\frac{3}{2} N k T_{vir} = -\frac{1}{2} \cdot -\frac{3}{5}\frac{GM^2}{R}$$

$$\cancel{\frac{3}{2}} N k T_{vir} = + \frac{\cancel{3}}{10} \frac{G M^2}{R}$$

$\uparrow$

want to get rid of N in this equation, so lets substitute $M^2 = M \cdot M = M \cdot (N \cdot \bar{m})$

$\uparrow$ Number of gas particles

$\uparrow$ average mass of a gas particle

$$\bar{m} \simeq \frac{m_p + m_e}{2} \simeq \frac{m_p}{2}$$

so $$\frac{\cancel{N}}{\cancel{2}} k T_{vir} = \frac{1}{10} \frac{G M \cancel{N}}{R} \frac{m_p}{\cancel{2}}$$

$$T_{vir} = \frac{G M_\odot m_p}{10 \, k R_\odot}$$

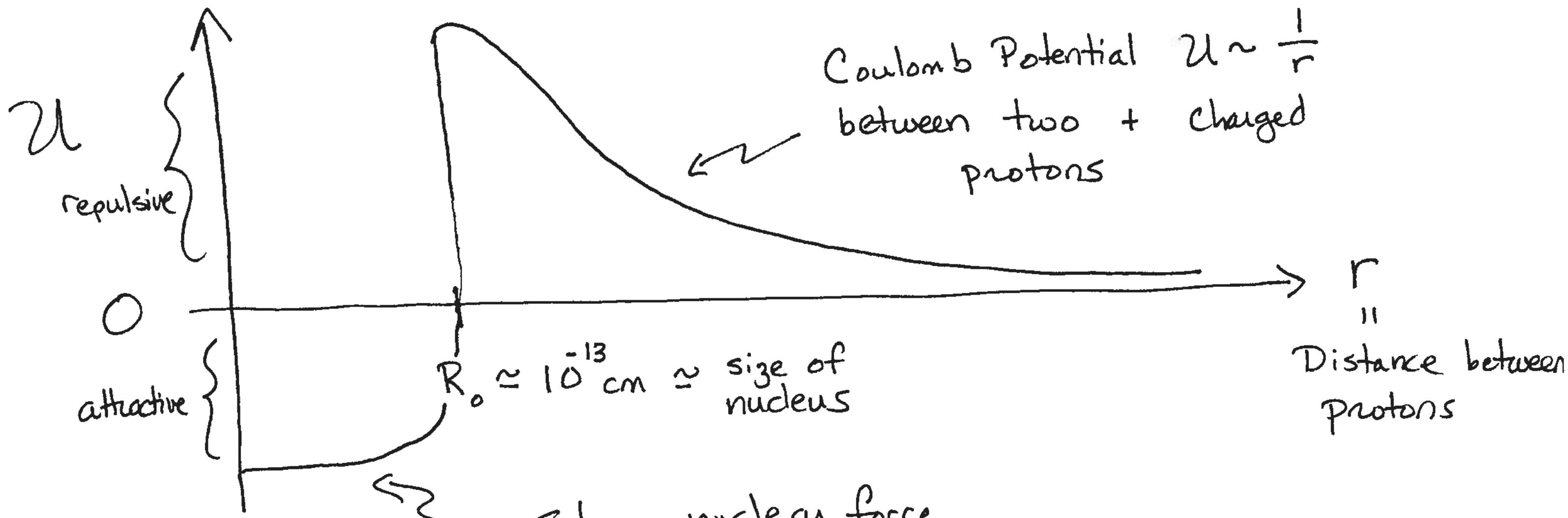
$$= \frac{(6.67 \times 10^{-8}) \ (2 \times 10^{33} g) \ (1.67 \times 10^{-24} g)}{10 \ (1.38 \times 10^{-16} \ erg \cdot K^{-1}) \ (7 \times 10^{10} cm)}$$

$$\simeq 2 \times 10^{6} \ K$$

Very Hot!

But is this hot enough for fusion?

Potential Energy Surface for Colliding Protons:



Strong nuclear force is attractive for $r < R_o$!

$$\mathcal{U}_{coulomb} = \frac{q^2}{r} \quad \leftarrow \quad q = \text{charge of proton} = 4.8 \times 10^{-10} \text{ esu (cgs units)} = 1.6 \times 10^{-19} \text{ C (MKS units)}$$

So the potential barrier that must be overcome is:

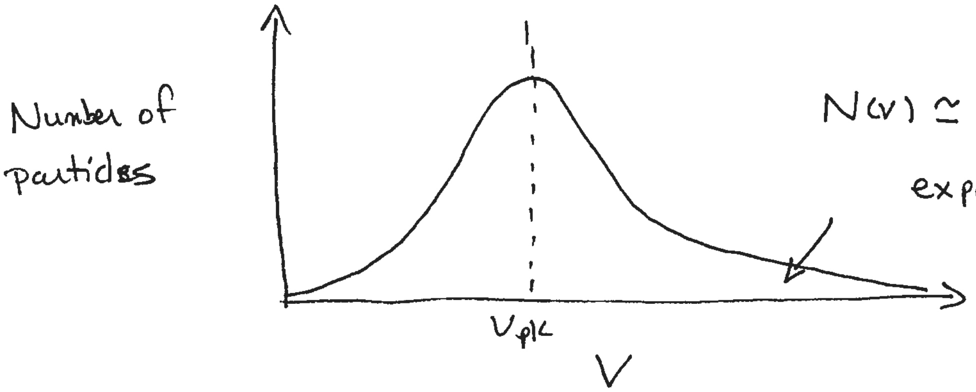
$$\mathcal{U}_{coulomb} = \frac{q^2}{R_o} = \frac{(4.8 \times 10^{-10} \text{ esu})^2}{10^{-13} \text{ cm}} \simeq 2 \times 10^{-6} \text{ erg}$$

OK, so how much kinetic energy is there is $T \sim 10^7$ K gas?

$$E_{Kinetic} = \frac{3}{2} k T = \frac{3}{2} (1.38 \times 10^{-16} \text{ erg} \cdot \cancel{K}^{-1}) \cdot 10^7 \cancel{K} \simeq 2 \times 10^{-9} \text{ erg}$$

$\Rightarrow \quad E_{Kinetic} \ll E_{Coulomb}$ by a factor of 1000!

IN reality, that is the average kinetic energy, but some particles have higher kinetic energies. A gas follows a Maxwellian Velocity Distribution:



$$N(v) \simeq e^{-mv^2/2kT} \simeq e^{-E_{kin}/kT}$$

exponential "tail" of proton energies.

So $N(v > \text{Coulomb barrier}) \sim e^{-E_{kin}/E_{coulomb}} \sim e^{-1000} \sim 10^{-430}$ (yikes!) not very many!

There are only $N = \dfrac{M_\odot}{m_H} = \dfrac{2 \times 10^{33}\ g}{2 \times 10^{-24}\ g/\text{proton}} \sim 10^{57}$ protons in the Sun!

NOT EVEN 1 proton in the sun @ $T \sim 10^7$ K has enough kinetic energy to over-come the electrostatic repulsion between protons!!

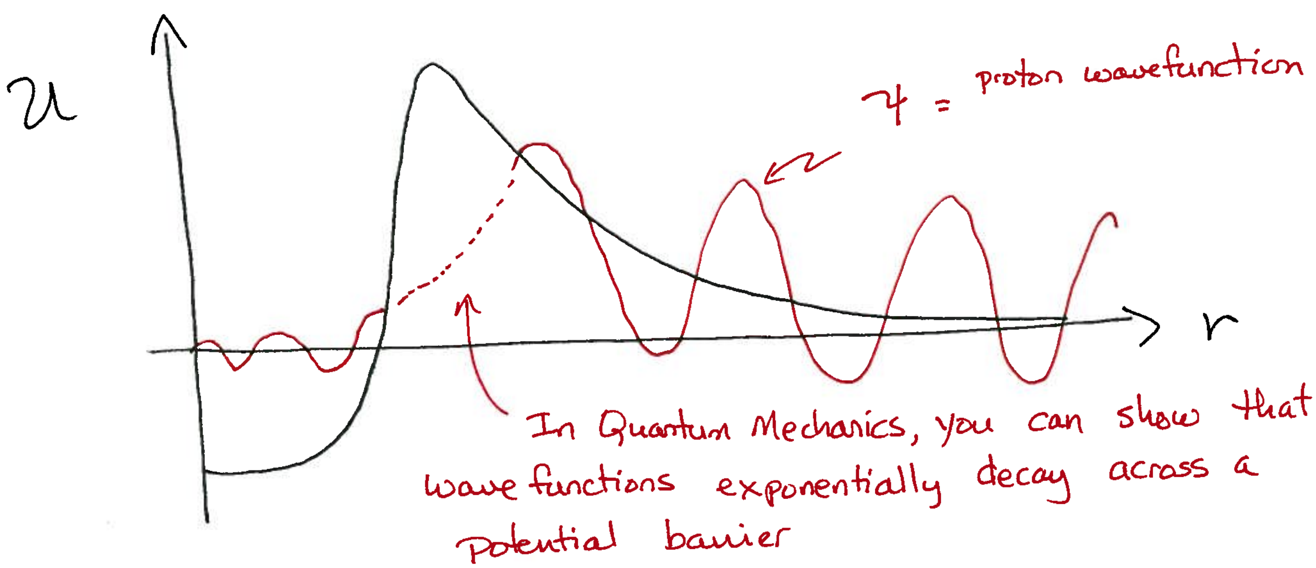
Classically, fusion should not occur!

The answer: Quantum Mechanics to the rescue!

"Quantum Tunneling"

According to Quantum Mechanics, particles are described by a wavefunction, ψ, and the probability of finding a particle at a position is space $\sim |\psi|^2$.

One property of wavefunctions is that they exist <u>everywhere</u> in space. <u>So</u>, there is some <u>non-zero</u> probability that the proton will "tunnel" through the Coulomb barrier.



The calculation of this probability is too advanced for this class – but it is high enough that it does occur!

The reaction is:

positron

electron neutrino

$$^1H + {}^1H \longrightarrow {}^2D + e^+ + \nu_e$$

Deuterium is just an isotope of hydrogen w/ 1 proton + 1 neutron

Wait a minute — we have 2 protons on the left side, but ^2D has 1 proton & 1 neutron. How did the proton convert into a neutron ??

β-decay $\quad p^+ \rightarrow n + e^+ + \nu_e$

this reaction is controlled by the nuclear "weak" force. It is very slow.

The timescale for a proton to (1) tunnel through the Coulomb barrier and (2) undergo β decay is $\simeq 10^{10}$ years in the Sun! Very Slow.

The p-p chain is how the Sun gets 91% of its energy:

p-p I:

$^1H + {}^1H \rightarrow {}^2D + e^+ + \nu_e$ $\quad \tau \sim 10^{10}$ years

$^2D + {}^1H \rightarrow {}^3He + \gamma$ (gamma ray photon) $\quad \tau \sim 6$ secs!

$^3He + {}^3He \rightarrow {}^4He + H + H$ $\quad \tau \sim 10^6$ years

$\Delta E_{tot} \simeq 26.2$ MeV (excludes neutrino energies since they don't interact strongly w/ matter in the Sun and escape easily)

Number of reactions in the Sun $= N_{reactions} = \dfrac{L_\odot}{\Delta E_{reaction}} = \dfrac{3.9 \times 10^{33}\ \text{erg} \cdot s^{-1}}{26.2 \times 10^6\ \cancel{eV} \cdot 1.602 \times 10^{-12}\ \text{erg}/\cancel{eV}}$

$\simeq 10^{38}\ s^{-1}$

so, even though 1 reaction is rare, there are 10^{57} protons in the Sun - so alot of reactions every second!