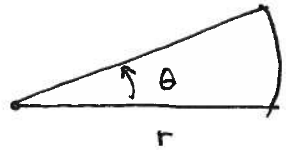
Photometric Concepts

SPRING 2010
AST 250

Defn. of a solid angle

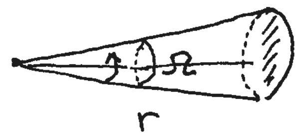
units

1 DIMENSION :



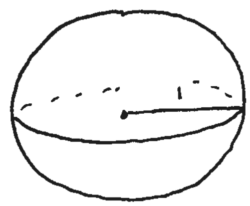
$\theta = \frac{S}{r}$ radians "rad"

2 DIMENSIONS :



$\Omega = \frac{A}{r^2}$ steradians "ster"

FOR a unit sphere

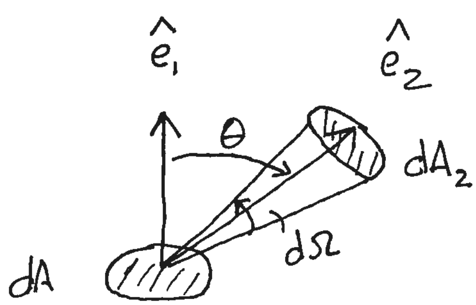


$$\Omega = \frac{\text{Area of Sphere}}{r^2} = \frac{4\pi \cancel{r^2}}{\cancel{r^2}} = 4\pi \text{ ster}$$

We define the <u>monochromatic specific intensity</u> I_ν or I_λ as the rate of Energy $\frac{dE_\nu}{dt}$ flowing through an area dA to another area dA_2 per solid angle $d\Omega$ in a frequency interval $[\nu, \nu + d\nu]$

$$I_\nu = \frac{dE}{dt \, \cos\theta \, dA \, d\Omega \, d\nu}$$

units: $\text{erg} \cdot s^{-1} \cdot cm^{-2} \cdot ster^{-1} \cdot Hz^{-1}$



$|\hat{e}_1| = |\hat{e}_2| = 1$

$\hat{e}_1 \cdot \hat{e}_2 = \cos\theta$

We could also define monochromatic specific intensity per wavelength interval $[\lambda, \lambda + d\lambda]$

$$I_\lambda = \frac{dE}{dt \; \cos\theta \, dA \; d\Omega \; d\lambda} \qquad \text{units: } erg \cdot s^{-1} \cdot cm^{-2} \cdot ster^{-1} \cdot cm^{-1}$$

How are I_ν and I_λ related?

$$\lambda \cdot \nu = c$$

$$\nu = \frac{c}{\lambda}$$

$$\frac{d\nu}{d\lambda} = -\frac{c}{\lambda^2}$$

$$d\nu = \frac{+c}{\lambda^2} d\lambda$$

$$\Rightarrow \quad I_\lambda = \frac{c}{\lambda^2} I_\nu$$

Monochromatic refers to radiation at a single λ or ν.

The total specific intensity

$$I = \int_0^\infty I_\nu \, d\nu \qquad erg \cdot s^{-1} \cdot cm^{-2} \cdot ster^{-1}$$

subscript always means "monochromatic"

We define the "Flux Density" as:

$$F = \int_{\substack{\text{Solid}\\\text{angle}}} I \cdot \cos\theta \, d\Omega \qquad erg \cdot s^{-1} \cdot cm^{-2}$$

= Rate of Energy flowing through dA from all solid angles.

Do not confuse "flux density" with "flux"

$$\text{Flux} = \mathcal{F} = \int_{\text{area}} F \, dA \qquad \underbrace{erg \cdot s^{-1}}$$

Flux has units of Power!

Remember 1 Watt = 10^{-7} $erg \cdot s^{-1}$

<u>Notice the units progression:</u>

Monochromatic specific intensity = I_ν $\qquad erg \cdot s^{-1} \cdot cm^{-2} \cdot ster^{-1} \cdot Hz^{-1}$

total specific intensity = $I = \int I_\nu \, d\nu$ $\qquad erg \cdot s^{-1} \cdot cm^{-2} \cdot ster^{-1}$

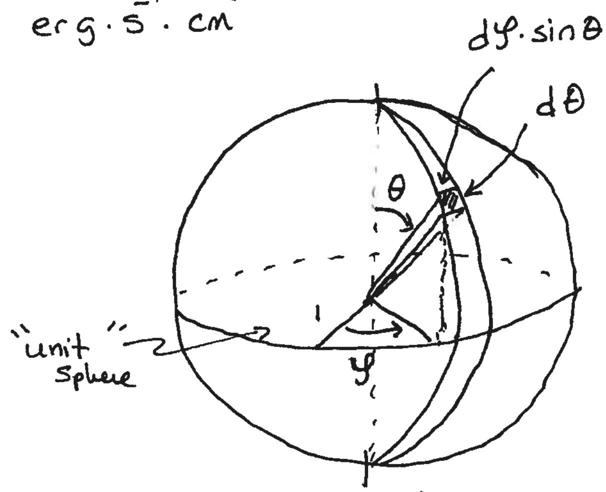
flux density = $F = \int I \cos\theta \, d\Omega$ $\qquad erg \cdot s^{-1} \cdot cm^{-2}$

flux = $\mathcal{F} = \int F \, dA$ $\qquad erg \cdot s^{-1}$

Let's calculate the total flux density for an "isotropic" radiation field

$$I(\hat{e}_2) = \text{CONSTANT} = I$$

Total Flux Density $\quad F = \int\limits_{\text{solid angle}} I \cos\theta \, d\Omega \quad erg \cdot s^{-1} \cdot cm^{-2}$



Spherical Coordinates

what is $d\Omega$?

$$dA = \sin\theta \, d\varphi \, d\theta$$

$$d\Omega = \frac{dA}{r^2} = \frac{dA}{1} = \sin\theta \, d\theta \, d\varphi$$

$$F = \int_{\theta=0}^{\pi} \int_{\varphi=0}^{2\pi} I \cos\theta \sin\theta \, d\theta \, d\varphi$$

$$= I \cdot \varphi \Big|_0^{2\pi} \cdot (-1) \cdot \int_{\theta=0}^{\pi} \cos\theta \, (-\sin\theta \, d\theta)$$

$$= 2\pi I \cdot \int_{\theta=\pi}^{0} \cos\theta \, d(\cos\theta)$$

$$= \cancel{2}\pi I \cdot \frac{1}{\cancel{2}} \cos^2\theta \Big|_{\pi}^{0} = \pi I \cdot \left(\cancelto{1}{\cos^2 0} - \cancelto{1}{\cos^2 \pi} \right)$$

$$\Rightarrow F = 0 \quad !!$$

This means there is no net flux density of radiation through dA $\Rightarrow$ Equal amounts flowing from all directions.

Therefore we define the emergent flux density

(For instance - the flux density emergent from a star's surface)

$$F^{+} = \int_{\theta=0}^{\pi/2} \int_{\varphi=0}^{2\pi} I \cos\theta \sin\theta \, d\theta \, d\varphi$$

$$F^{+} = \pi I \cdot \cos^2\theta \Big|_{\pi/2}^{0} = \pi I \cdot \left(\overset{1}{\cancel{\cos^2}\, 0} - \overset{0}{\cancel{\cos^2}\, \pi/2} \right)$$

$$F^{+} = \pi I$$

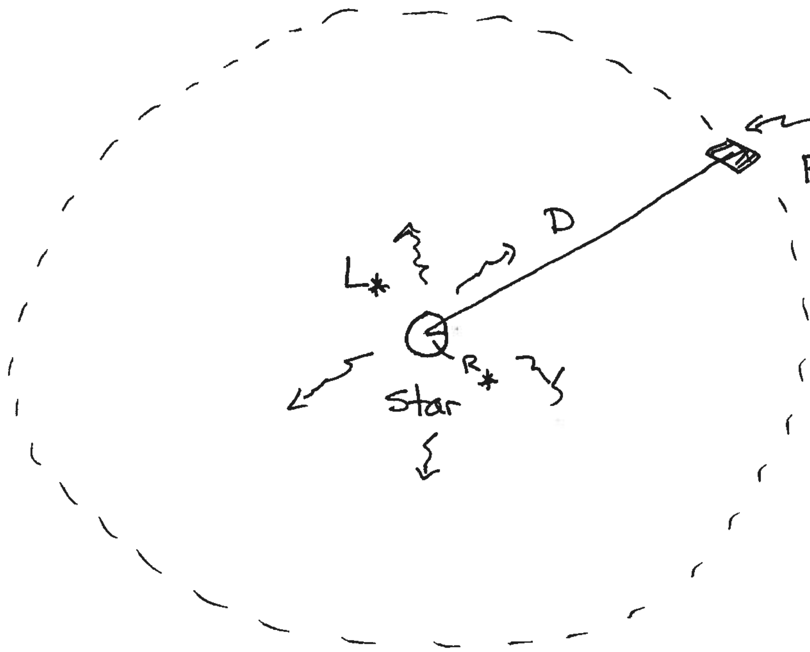
also called the "astrophysical" flux density.

So for a star, the Luminosity is just the integrated total emergent flux density from the star

$$L = \underset{\text{Surface area of star}}{\int} F_{*}^{+} \, dA = 4\pi R_{*}^{2} F_{*}^{+} \quad \underset{\text{units of power}}{\mathrm{erg} \cdot s^{-1}}$$

Luminosity = total flux emerging from star.

Conservation of Energy



$F_{obs}(D)$ = observed flux density at distance D
$\mathrm{erg} \cdot s^{-1} \cdot \mathrm{cm}^{-2}$

$$L_* = 4\pi R_*^2 F_*^+ \underset{\text{energy is conserved}}{=\!=} 4\pi D^2 F_{obs}(D)$$

$$\Rightarrow \quad F_{obs}(D) = F_*^+ \cdot \left(\frac{R_*}{D}\right)^2 \sim \frac{1}{D^2}$$

This is the derivation of the inverse square law!

Now, what about the specific intensity?

$$F \sim I \cdot \Omega \quad \Rightarrow \quad I \sim \frac{F}{\Omega}$$

$F \sim \frac{1}{D^2}$ and $\Omega \sim \frac{1}{D^2}$ from defn. of solid angle

$$\Rightarrow I \sim \frac{\frac{1}{D^2}}{\frac{1}{D^2}} \sim \text{CONST} \Rightarrow$$

Specific intensity is not distance dependent!