Blackbody Radiation

ASTR 250
SPRING 2010

A blackbody does not reflect or scatter light, but absorbs and remits light completely.

Planck's Law described the specific intensity of a blackbody:

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \cdot \frac{1}{e^{h\nu/kT} - 1} \quad erg \cdot s^{-1} \cdot cm^{-2} \cdot ster^{-1} \cdot Hz^{-1}$$

$$B_\lambda(T) = \frac{c}{\lambda^2} B_\nu(T) \quad \text{by definition} \quad \left(\begin{array}{c} \text{Remember} \\ d\nu \neq d\lambda \\ \text{but } d\nu = -\frac{c}{\lambda^2} d\lambda \end{array} \right)$$

$$B_\lambda(T) = \frac{\cancel{c}}{\lambda^2} \cdot \frac{2h\cancel{\nu^3} c^3}{\cancel{c^2} \lambda^3} \cdot \frac{1}{e^{hc/\lambda kT} - 1}$$

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5} \cdot \frac{1}{e^{hc/\lambda kT} - 1} \quad erg \cdot s^{-1} \cdot cm^{-2} \cdot ster^{-1} \cdot cm^{-1}$$

The total intensity of a blackbody:

$$B(T) = \int_0^\infty B_\nu(T)\, d\nu = a \cdot T^4$$

we need the theory of complex variable to evaluate this integral!

$$a = \text{"radiation" constant} = \frac{2k^4\pi^4}{c^2h^3\,15}$$

Remember, the astrophysical flux density (or emergent flux density) is

$$F^+ = \pi \cdot B(T) = \pi \cdot a T^4$$
$$= \sigma T^4$$

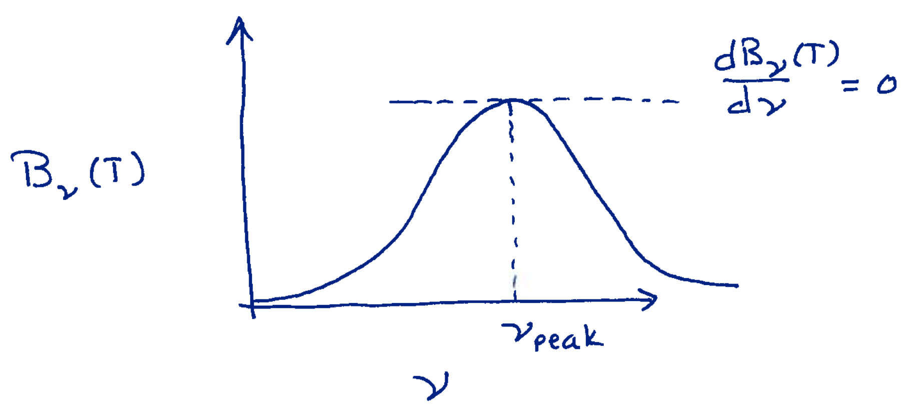
$$\sigma = \text{Stephan-Boltzmann Constant} = 5.67 \times 10^{-5}\ erg \cdot s^{-1} \cdot cm^{-2} \cdot K^{-4}$$

For a star

$$L_* = 4\pi R_*^2 F_*^+ = \boxed{4\pi R_*^2 \sigma T_{eff}^4}$$

$T = T_{eff}$ is the "effective" temperature of a star described by a blackbody spectrum.

How can we characterize the shape of the blackbody curve?



Wein's Law:

$$\frac{dB_\lambda(T)}{d\lambda} = 0 \Rightarrow \lambda_{max} \cdot T = 0.2898 \text{ cm} \cdot K$$

$$\frac{dB_\nu(T)}{d\nu} = 0 \Rightarrow \frac{\nu_{max}}{T} = 5.879 \times 10^{10} \text{ Hz} \cdot K^{-1}$$

__NOTE__: $\nu_{max} \cdot \lambda_{max} \neq c$ because $d\nu \neq d\lambda$!

The two curves are different $B_\nu(T)$ and $B_\lambda(T)$.

Wein's Law example :

The coldest brown dwarf discovered has $T_{eff} = 620\,K$

$$\Rightarrow \quad \lambda_{max} = \frac{0.2898\ cm\cdot\cancel{K}}{620\ \cancel{K}} = 4.7\,\mu m$$ <- Peaks in the near-IR!

Did they discover this object @ visible wavelengths? NO.

Cosmic Microwave background :

$$T_{cmb} = 2.725\ K$$

$$\Rightarrow \quad \lambda_{max} = \frac{0.2898\ cm\cdot\cancel{K}}{2.725\ \cancel{K}} = 1.06\ mm$$

$$\nu_{max} = 5.879\times10^{10}\ Hz\cdot\cancel{K^{-1}}\cdot 2.725\ \cancel{K} = 160\ GHz$$

N.B. ~~λ_{max}~~ if we find $\lambda = \frac{c}{\nu_{max}} = 1.9\ mm \neq 1.06\ mm = \lambda_{max}$!!

Limits of the Planck function:

Notice that $h\nu/k$ has units of T.

1. $h\nu/kT \gg 1$ "Wein Approximation"

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \cdot \frac{1}{e^{h\nu/kT} - 1} \xrightarrow[h\nu/kT \gg 1]{} \frac{2h\nu^3}{c^2} \cdot e^{-h\nu/kT}$$

exponential dependance

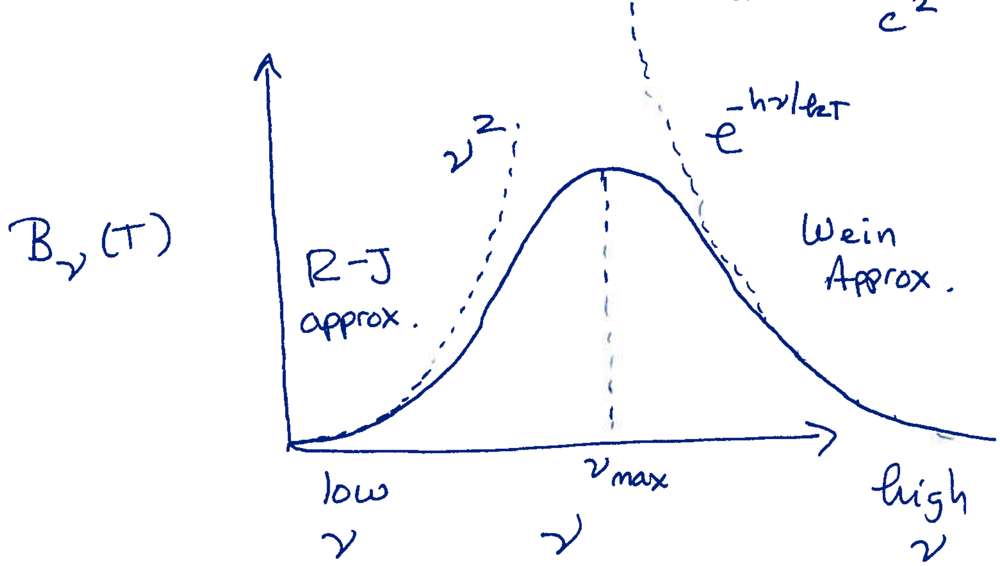
2. $h\nu/k \ll T$ "Rayleigh-Jeans Approximation"

$$e^{h\nu/kT} \simeq 1 + h\nu/kT + \ldots$$ (Taylor expansion)

so
$$B_\nu(T) = \frac{2h\nu^3}{c^2} \cdot \frac{1}{e^{h\nu/kT} - 1} \xrightarrow[h\nu/k \ll T]{} \frac{2h\nu^3}{c^2} \cdot \frac{1}{\cancel{1} + \frac{h\nu}{kT} - \cancel{1}}$$

$$= \frac{2\cancel{h}\nu^{\cancel{3}\,2}}{c^2} \cdot \frac{kT}{\cancel{h}\cancel{\nu}}$$

$$= \frac{2kT\nu^2}{c^2}$$ depends on ν^2



Example :

Are we in the R-J limit if we are observing a 100 K or a 10 K blackbody with a radio telescope at $\lambda = 1\,mm$?

$$\frac{h\nu}{k} = \frac{hc}{\lambda k} = \frac{6.67 \times 10^{-27}\, \cancel{erg} \cdot \cancel{s} \cdot 3 \times 10^{10}\, \cancel{cm} \cdot \cancel{s^{-1}}}{1.0 \times 10^{-1}\, \cancel{cm} \cdot 1.381 \times 10^{-16}\, \cancel{erg} \cdot K^{-1}} = 14.4\ K$$

So $\quad T = 100\,K \gg h\nu/k \quad \Rightarrow$ Yes.

$\quad T = 10\,K < h\nu/k \quad \Rightarrow$ NO.

How are black bodies related to magnitudes ?

We define the "bolometric correction" BC

$$m_{bol} = m_V - BC$$

$BC \equiv 0$ For an F5V stellar type

Then $\quad M_{bol} - M_{bol,\odot} = -2.5 \log_{10} \frac{L}{L_\odot}$

See slides for a table of BCs for different types of stars.

Also tabulated in Allen's Astrophysical Quantities.