## AST 300B - Spring 2017 In-class/take-home Problems Due: Wednesday January 23

4. The brightness temperature, $\mathrm{T}_{\mathrm{B}}$, of a source is the temperature that is directly proportional to the observed intensity given by the Planck function, $\mathrm{B}_{v}\left(\mathrm{~T}_{\mathrm{B}}\right)$, in the Rayleigh-Jeans limit ( $\mathrm{hv} / \mathrm{kT}_{\mathrm{B}} \ll 1$ ). Radio telescope don't actually measure specific intensity, but instead measure the flux density of the source observed within the diffraction beam of the telescope with solid angle $\Omega=\pi \theta^{2}$ fwhm $/ 4 \ln (2)$ for a FWHM "beam width" of $\theta_{\text {fwhm }}$ (see Figure below). Derive the equation for how $\mathrm{T}_{\mathrm{B}}$ is related to observed flux density $\mathrm{F}_{v}$ assuming $\mathrm{T}_{\mathrm{B}}$ is constant across the telescope beam solid angle (write your answer in terms of $\theta_{\text {fwhm }}$ ). This expression is very handy for going from K to Janskys (Jansky is the unit of flux density used in radio astronomy: $1 \mathrm{Jy}=10^{-23} \mathrm{erg} \mathrm{s}^{-1} \mathrm{~cm}^{-2} \mathrm{~Hz}^{-1}$ ). Hint: radio telescope FWHM are usually small angles - for instance, $\theta_{\text {fwhm }}=67$ " for the Kitt Peak 12 m at 90 GHz .


Figure 1: A radio telescope diffraction pattern. This "power pattern" or "beam pattern" is the sensitivity of the telescope to radiation coming from different directions. The angle that corresponds to where the sensitivity drops from 1.0 to 0.5 in the main lobe is called the FWHM. Radio telescope main lobes are usually well characterized by a Gaussian function of angle $\theta$.
5. Protostars are classified (Class $0 / \mathrm{I} / \mathrm{II} / \mathrm{III}$ ) by how deeply embedded they are within the dusty cores in which they form within molecular clouds. One evolutionary metric is the "Bolometric Temperature" ( $\mathrm{T}_{\text {bol }}$ ) which is defined as the temperature of the blackbody that has the same mean frequency as the observed Spectral Energy Distribution (SED) of the protostar. In a seminal paper by Chen et al. 1995 ApJ 445, 377, $\mathrm{T}_{\text {bol }}$ was determined for protostars in different evolutionary phases. Derive their equation:

$$
T_{\mathrm{bol}}=\frac{\zeta(4)}{4 \zeta(5)} \frac{h\langle v\rangle}{k}=1.25 \times 10^{-11}\langle v\rangle \mathrm{K} \mathrm{~Hz}^{-1}
$$

Hint: To calculate the mean (average) frequency of a Planck function, think about the continuous analogy to weighted averaging.

-Bolometric luminosity-temperature diagrams of all 235 YSOs in the three regions. The open circles are the sources with a known spectral type, and the les are the sources without a known spectral type. The solid line shows the zero-age main sequence for stellar mass of $0.4-6 M_{\odot}$. Sources without peaks in tra are shown with the arrows. The dashed lines show the regimes approximately corresponding to the YSO spectral energy distribution classes 0 , I, II,

