

## AST 300B – Spring 2019

### In-class/take-home Problems Due: Wednesday April 17th

35. Hyperfine splitting of energy levels occurs because of the interactions between the spin of the nucleus and another angular momentum in the atom or molecule.

(a) In the most common molecular case, the spin of the nucleus couples with the rotational angular momentum of the molecule (denoted  $J$ ). Let's consider the HCN molecule where the nuclear spin of  $^{14}\text{N}$  is  $I_{\text{nuc}} = 1$  (ignore the spin of the H in this problem). What are the possible scalar values of the total angular momentum (denoted  $F$ ) from the vector addition of the spin of the nucleus and the rotational angular momentum of the molecule in the ground state  $J = 0$  and in the first excited state  $J = 1$ ? Given the selection rules for electric dipole transitions require that  $\Delta F = 0$  (but NOT 0-0), +1, or -1, explain why the HCN  $J = 1-0$  rotational transition has 3 spectral lines instead of just 1 spectral line (see Figure below for an example HCN 1-0 spectrum taken at the ARO 12m telescope).

(b) The theoretical hyperfine intensity ratios for the HCN 1-0 lines are given by the statistical weights of the upper energy levels. In the optically thin limit, we should observe the hyperfine lines to have those ratios. What are the optically thin hyperfine line ratios? Eyeballing the spectrum below, are these lines optically thin?

