## AST 300B – Spring 2019 In-class Problem Due: Fri. Feb. 15

17. Let's calculate the long wavelength "spectral index" of dust emission. The spectral index,  $\alpha$ , is defined as the power-law slope of the observed flux density over some range of wavelengths:  $F_{\nu} \sim \nu^{\alpha}$ . Consider an **optically thin** source of dust emission with emission well described by a source function  $B_{\nu}(T_d)$  over a solid angle  $\Omega$ , and dust mass opacity parametrized by  $\kappa(\nu) = \kappa_0 (\nu/\nu_0)^{\beta}$  (cm<sup>2</sup>/g of dust) at long wavelengths. Assume that the source is small and  $F_{\nu} = I_{\nu} \Omega$  is a good approximation.

(a) How does the dust optical depth  $\tau_{\nu}$  depend on frequency?

(b) In the Rayleigh-Jeans limit at long wavelength what is the spectral index  $\alpha$  for (1) blackbody emission and (2) for optically thin dust emission with a dust opacity index of  $\beta$ ?

