Synchrotron Emission

ASTR 300B

Typical spiral galaxy @ 1 GHz ~ 10% free-free and 90% synchrotron $\Rightarrow$ Synchrotron emission dominated at low ν.

3 types of Magnetobremsstrahlung:

Gyro-radiation	$KE \ll m_e c^2$	$(v \ll c)$
Cyclotron	$KE \sim m_e c^2 \sim 511$ keV	
Synchrotron	$KE \gg m_e c^2$	(typically ultra-relativistic)

Lorentz Force is $\perp$ to e^- velocity

$$\vec{F} = \frac{e(\vec{v} \times \vec{B})}{c} \qquad ma = m\omega^2 r = \frac{e}{c}\omega r B$$

Define gyro frequency: $\omega_G \equiv 2\pi\nu_G \equiv \frac{eB}{mc}$

$$\nu_G \simeq 2.8\ \mathrm{MHz}\left(\frac{B}{1\ \mathrm{Gauss}}\right) \Leftarrow$$

typical ISM $B \sim 10\,\mu G$

ν_G too low to propagate

$(< \nu_{plasma})$

$\Rightarrow$ Synchrotron emission due to e^-/e^+ dominates.

Relativistic effects must be accounted for:

$$E = \gamma m_e c^2, \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{Lorentz factor}, \ \beta = \frac{v}{c}$$

Relativistic Larmor Formula: $P(t) = \frac{2}{3}\frac{e^2}{c^3}\gamma^4(a_\perp^2 + \gamma^2 a_\parallel^2)$ erg$\cdot$s^{-1} (cgs version)

But $a_\perp$ dominates and $a_\perp = \frac{eB\cancel{c}\beta\sin\alpha}{\gamma m_e \cancel{c}} \leftarrow$ "pitch angle"

$$\Rightarrow P \sim \frac{e^4 B^2}{m_e^2 c^3}\gamma^2$$

Also, radiation is "beamed" into narrow cone with $\theta \sim \frac{1}{\gamma}$

Just like for bremsstrahlung, we calculate $P(\nu)$. This is very tedious (see Longair book: High Energy Astrophysics, Ch. 8).

For a <u>single</u> e^-:

$$P(\nu; E) = \frac{\sqrt{3}\, e^2 B \sin\alpha}{m_e c^2} \left(\frac{\nu}{\nu_c}\right) \int_{\nu/\nu_c}^{\infty} K_{5/3}(x)\, dx$$

another modified Bessel Fctn of 2nd kind

$$\nu_c = \text{Critical Frequency} \equiv \frac{3}{2} \gamma^2 \nu_G \sin\alpha$$

Power spectrum for single e^- peaks close to ν_c and exponentially falls off $\nu > \nu_c$

Synchrotron emission is typically <u>non-thermal</u> $\Rightarrow$ e^- do <u>NOT</u> follow Maxwell-Boltzmann Distribution in v or E. Instead they typically follow a power-law distribution:

$$N_{e^-}(E) = N_0\, E^{-\Gamma} \quad cm^{-3} \cdot erg^{-1}$$

$$\int_{E_{min}}^{E_{MAX}} N_{e^-}(E)\, dE = n_{e^-}$$

number density of <u>relativistic</u> e^-

$$j_\nu = \frac{1}{4\pi} \int P(\nu, E)\, N(E)\, dE$$

$ster^{-1} \quad erg\, s^{-1} \cdot Hz^{-1} \quad cm^{-3} \cdot \cancel{erg}^{-1} \cdot \cancel{erg}$ = units of j_ν ✓

For $\Gamma > 1/3$ and $E_{min} \lesssim E \lesssim E_{max}$ we have numerical solutions:

$$j_\nu = C_5(\Gamma)\, N_0\, B_\perp^{(\Gamma+1)/2} \left(\frac{\nu}{2C_1}\right)^{-\frac{(\Gamma-1)}{2}} \quad erg \cdot s^{-1} \cdot cm^{-3} \cdot ster^{-1} \cdot Hz^{-1}$$

$$\alpha_\nu = C_6(\Gamma)\, N_0\, B_\perp^{(\Gamma+2)/2} \left(\frac{\nu}{2C_1}\right)^{-\frac{(\Gamma+4)}{2}} \quad cm^{-1}$$

C_1, C_5, C_6 are Pacholczyk's consts $\quad C_1 = 6.27 \times 10^{18}$ Hz

$B_\perp$ is B field $\perp$ to line of sight $\Rightarrow$ 2D in plane of sky

Why $j_\nu \sim \nu^{-\frac{(\Gamma-1)}{2}}$?

Since most power comes out at $\nu \approx \nu_c$ we can associate each E with ν_c

$j_\nu \sim P(\nu)\, N(E)\, dE$

- $P(\nu) \sim \nu$
- $\nu_c \sim \gamma^2 \sim E^2$ (also $E = \gamma m_e c^2 \Rightarrow E \sim \gamma$)
- $\Rightarrow E \sim \nu^{+1/2}$
- $\Rightarrow \frac{dE}{d\nu} \sim \nu^{-1/2}$

$j_\nu \sim \nu \cdot \nu^{-\Gamma/2} \cdot \nu^{-1/2}$

$\sim \nu^{-(\Gamma-1)/2}$