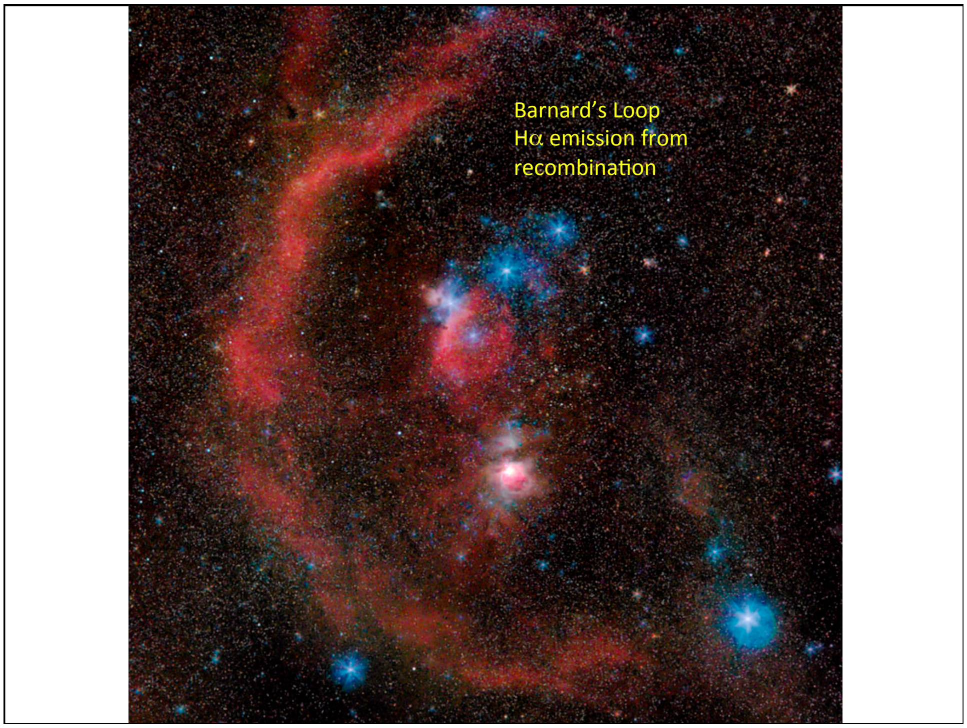
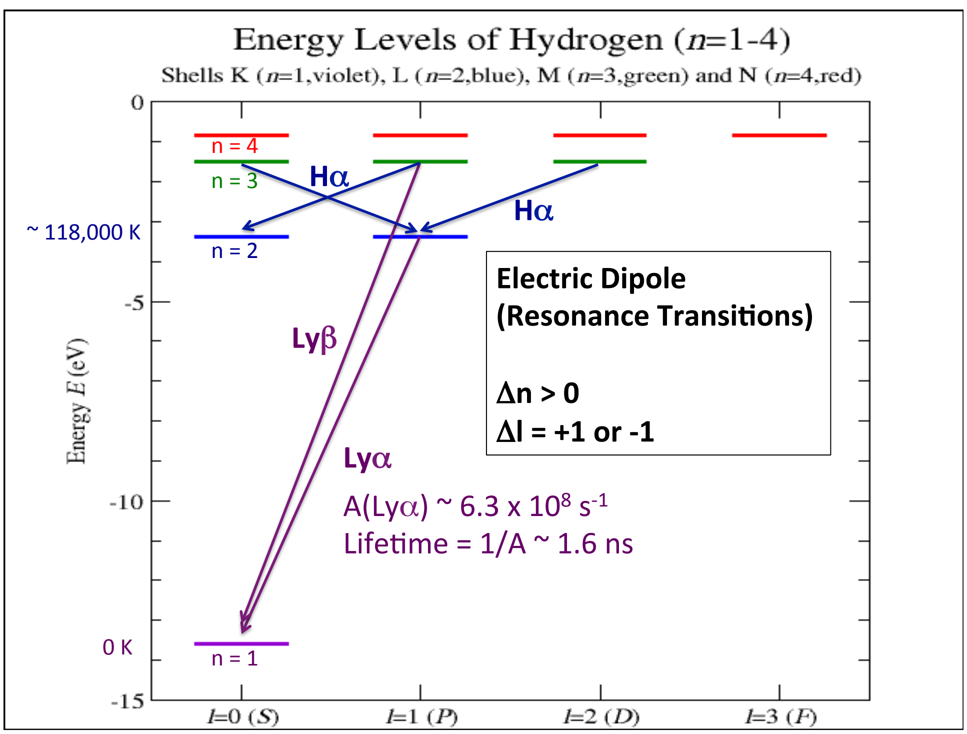
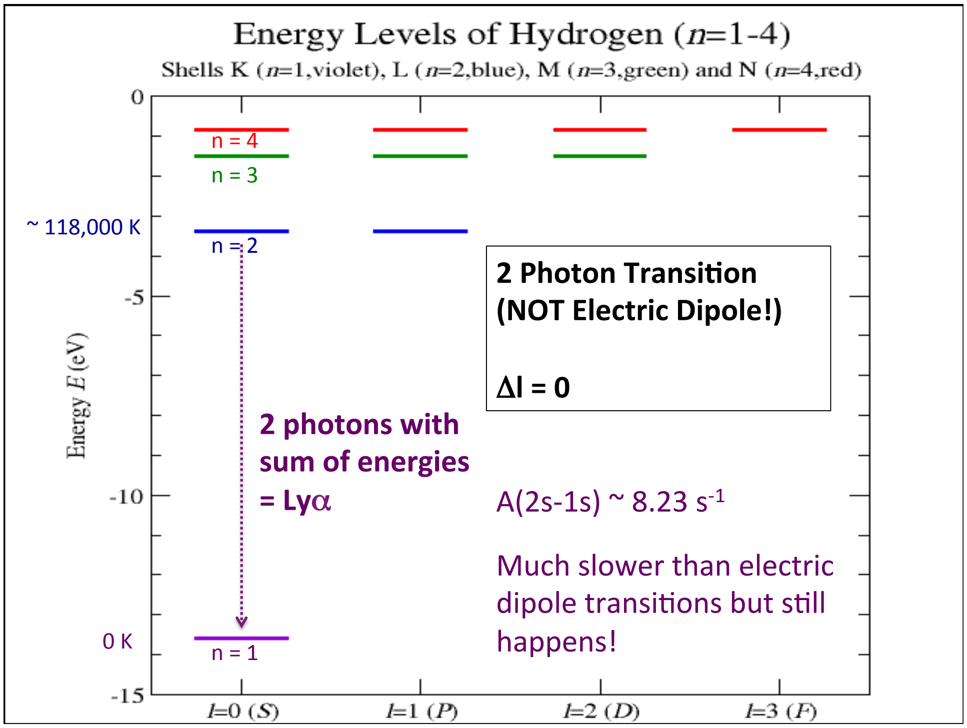
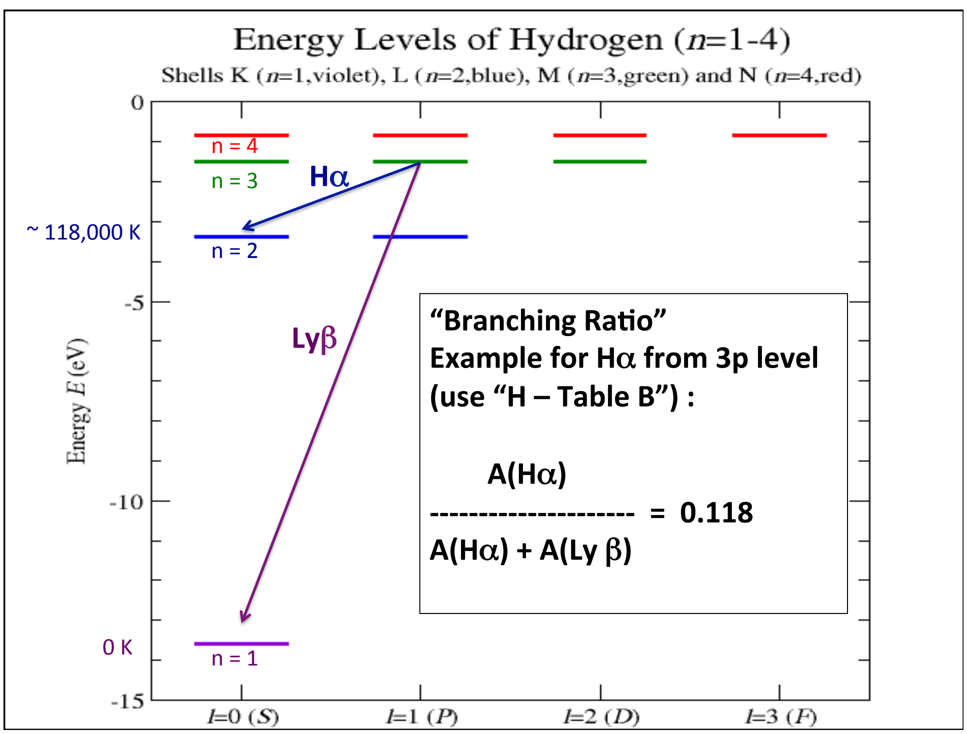
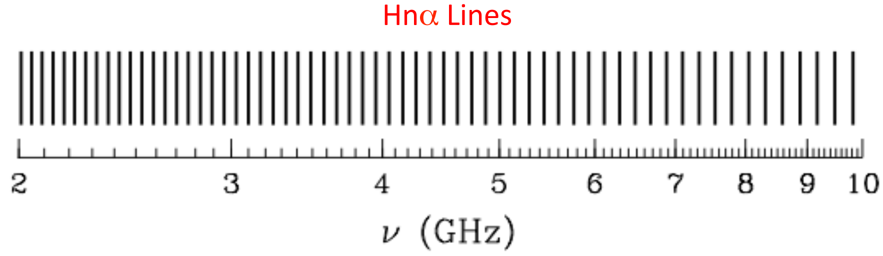

Barnard's Loop
Hα emission from
recombination


Energy Levels of Hydrogen (n=1-4)
Shells K (n=1,violet), L (n=2,blue), M (n=3,green) and N (n=4,red)
0
-5
-10
-15
Energy E (eV)
n = 4
n = 3
n = 2
n = 1
~ 118,000 K
0 K
Hα
Hα
Lyβ
Lyα
A(Lyα) ~ 6.3 x 10^8 s^-1
Lifetime = 1/A ~ 1.6 ns
Electric Dipole
(Resonance Transitions)
Δn > 0
Δl = +1 or -1
l=0 (S)
l=1 (P)
l=2 (D)
l=3 (F)


Energy Levels of Hydrogen (n=1-4)
Shells K (n=1,violet), L (n=2,blue), M (n=3,green) and N (n=4,red)
Energy E (eV)
0
-5
-10
-15
~ 118,000 K
0 K
n = 4
n = 3
n = 2
n = 1
l=0 (S)
l=1 (P)
l=2 (D)
l=3 (F)
2 Photon Transition
(NOT Electric Dipole!)
Δl = 0
2 photons with
sum of energies
= Lyα
A(2s-1s) ~ 8.23 s-1
Much slower than electric
dipole transitions but still
happens!


Energy Levels of Hydrogen (n=1-4)
Shells K (n=1,violet), L (n=2,blue), M (n=3,green) and N (n=4,red)
Energy E (eV)
0
-5
-10
-15
~ 118,000 K
0 K
n = 4
n = 3
n = 2
n = 1
l=0 (S)
l=1 (P)
l=2 (D)
l=3 (F)
Hα
Lyβ
"Branching Ratio"
Example for Hα from 3p level
(use "H – Table B") :
A(Hα)
-------------------- = 0.118
A(Hα) + A(Ly β)

Radio Recombination Lines

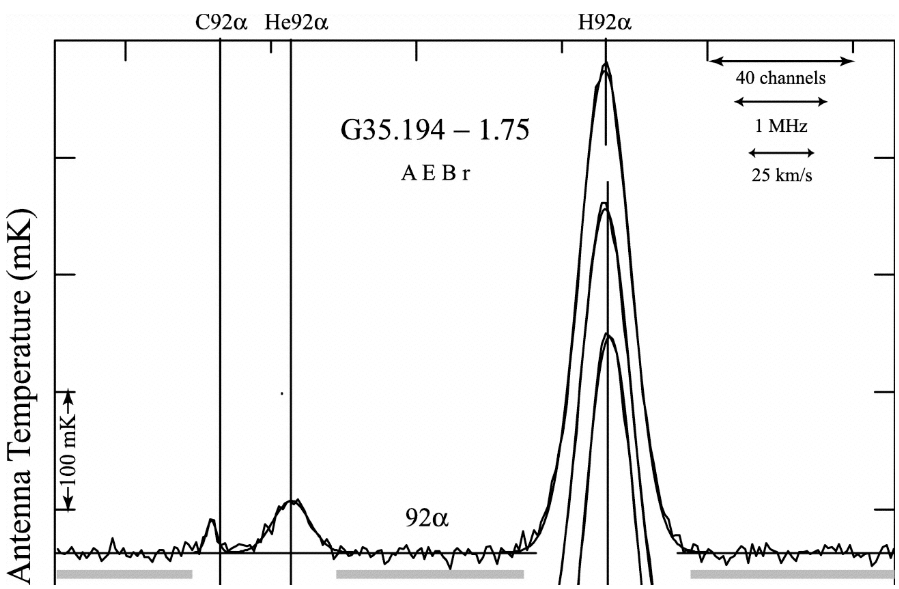


Often, multiple radio recombination lines can be observed within the same bandpass in the radio. All of the $\Delta n = -1$ H recombination lines between 2 GHz and 10 GHz are shown above.

$A_{n+1\text{-}n} \sim 6.130 \times 10^9 (n + 0.7)^{-5}\ s^{-1}$

good to within 1% n >= 4 (Menzel 1968)

Radio Recombination Lines



Quireza et al. 2006 ApJS 165 338

H Recombination Rates

Table 14.1 Recombination Coefficients $\alpha_{n\ell}$ ($cm^3 s^{-1}$) for H.[a] The approximation formulae are valid for $0.3 \lesssim T_4 \lesssim 3$. For a broader range of T, see Eq. (14.5,14.6).

$\alpha_n(^2L)$	Temperature T: 5×10^3 K	1×10^4 K	2×10^4 K	approximation
α_{1s}	2.28×10^{-13}	1.58×10^{-13}	1.08×10^{-13}	$1.58 \times 10^{-13} T_4^{-0.539-0.014 \ln T_4}$
α_{2s}	3.37×10^{-14}	2.34×10^{-14}	1.60×10^{-14}	$2.34 \times 10^{-14} T_4^{-0.537-0.016 \ln T_4}$
α_{2p}	8.33×10^{-14}	5.35×10^{-14}	3.24×10^{-14}	$5.35 \times 10^{-14} T_4^{-0.681-0.061 \ln T_4}$
α_2	1.17×10^{-13}	7.69×10^{-14}	4.84×10^{-14}	$7.69 \times 10^{-14} T_4^{-0.637-0.045 \ln T_4}$
α_{3s}	1.13×10^{-14}	7.81×10^{-15}	5.29×10^{-15}	$7.81 \times 10^{-15} T_4^{-0.548-0.021 \ln T_4}$
α_{3p}	3.17×10^{-14}	2.04×10^{-14}	1.23×10^{-14}	$2.04 \times 10^{-15} T_4^{-0.683-0.068 \ln T_4}$
α_{3d}	3.03×10^{-14}	1.73×10^{-14}	9.09×10^{-15}	$1.73 \times 10^{-14} T_4^{-0.869-0.087 \ln T_4}$
α_3	7.33×10^{-14}	4.55×10^{-14}	2.67×10^{-14}	$4.55 \times 10^{-14} T_4^{-0.729-0.059 \ln T_4}$
α_{4s}	5.23×10^{-15}	3.59×10^{-15}	2.40×10^{-15}	$3.59 \times 10^{-15} T_4^{-0.562-0.028 \ln T_4}$
α_{4p}	1.51×10^{-14}	9.66×10^{-15}	5.81×10^{-15}	$9.66 \times 10^{-15} T_4^{-0.689-0.064 \ln T_4}$
α_{4d}	1.90×10^{-14}	1.08×10^{-14}	5.68×10^{-15}	$1.08 \times 10^{-14} T_4^{-0.871-0.081 \ln T_4}$
α_{4f}	1.09×10^{-14}	5.54×10^{-15}	2.56×10^{-15}	$5.54 \times 10^{-15} T_4^{-1.045-0.099 \ln T_4}$
α_4	5.02×10^{-14}	2.96×10^{-14}	1.65×10^{-14}	$2.96 \times 10^{-14} T_4^{-0.803-0.059 \ln T_4}$
α_A	6.82×10^{-13}	4.18×10^{-13}	2.51×10^{-13}	$4.18 \times 10^{-13} T_4^{-0.721-0.021 \ln T_4}$
α_B	4.54×10^{-13}	2.59×10^{-13}	1.43×10^{-13}	$2.59 \times 10^{-13} T_4^{-0.833-0.034 \ln T_4}$

[a] From Hummer & Storey (1987)

H Recombination

Baker & Menzel 1938

CASE A : Optically thin to ionizing photons

→ EVERY ionizing photon emitted during recombination escape

CASE B : Optically thick to radiation just above 13.6 eV

→ ALL ionizing photons are re-absorbed. Recombination to n=1 do not affect ionization balance – only recombinations to n >= 2 reduce ionization

Table 14.2 Case B Hydrogen Recombination Spectrum[a] for $n_e = 10^3$ cm^{-3}

	T(K) 5000	10,000	20,000
α_B(cm^3 s^{-1})	4.53×10^{-13}	2.59×10^{-13}	1.43×10^{-13}
$\alpha_{eff,2s}/\alpha_B$	0.305	0.325	0.356
$\alpha_{eff,H\alpha}$(cm^3 s^{-1})	2.20×10^{-13}	1.17×10^{-13}	5.96×10^{-14}
$\alpha_{eff,H\beta}$(cm^3 s^{-1})	5.40×10^{-14}	3.03×10^{-14}	1.61×10^{-14}
$4\pi j_{H\beta}/n_e n_p$(erg cm^3 s^{-1})	2.21×10^{-25}	1.24×10^{-25}	6.58×10^{-26}
Balmer-line intensities relative to Hβ 0.48627 μm			
$j_{H\alpha\ 0.65646}/j_{H\beta}$	3.03	2.86	2.74
$j_{H\beta\ 0.48627}/j_{H\beta}$	1.	1.	1.
$j_{H\gamma\ 0.43418}/j_{H\beta}$	0.459	0.469	0.475
$j_{H\delta\ 0.41030}/j_{H\beta}$	0.252	0.259	0.264
$j_{H\epsilon\ 0.39713}/j_{H\beta}$	0.154	0.159	0.163
$j_{H8\ 0.38902}/j_{H\beta}$	0.102	0.105	0.106
$j_{H9\ 0.38365}/j_{H\beta}$	0.0711	0.0732	0.0746
$j_{H10\ 0.37990}/j_{H\beta}$	0.0517	0.0531	0.0540
Paschen ($n \to 3$) line intensities relative to corresponding Balmer lines			
$j_{P\alpha\ 1.8756}/j_{H\beta}$	0.405	0.336	0.283
$j_{P\beta\ 1.2821}/j_{H\gamma\ 0.43418}$	0.399	0.347	0.305
$j_{P\gamma\ 1.0941}/j_{H\delta\ 0.41030}$	0.391	0.348	0.311
$j_{P\delta\ 1.0052}/j_{H\epsilon\ 0.39713}$	0.386	0.348	0.314
$j_{P\epsilon\ 0.95487}/j_{H8\ 0.38902}$	0.382	0.348	0.316
$j_{P9\ 0.92317}/j_{H9\ 0.38365}$	0.380	0.347	0.317
$j_{P10\ 0.90175}/j_{H10\ 0.37990}$	0.380	0.347	0.317
Brackett ($n \to 4$) line intensities relative to corresponding Balmer lines			
$j_{Br\alpha\ 4.0523}/j_{H\gamma\ 0.43418}$	0.223	0.169	0.131
$j_{Br\beta\ 2.6259}/j_{H\delta\ 0.41030}$	0.219	0.174	0.141
$j_{Br\gamma\ 2.1661}/j_{H\epsilon\ 0.39713}$	0.212	0.174	0.144
$j_{Br\delta\ 1.9451}/j_{H8\ 0.38902}$	0.208	0.173	0.145
$j_{Br\epsilon\ 1.8179}/j_{H9\ 0.38365}$	0.204	0.173	0.146
$j_{Br10\ 1.7367}/j_{H10\ 0.37990}$	0.202	0.172	0.146
Pfundt ($n \to 5$) line intensities relative to corresponding Balmer lines			
$j_{65\ 7.4599}/j_{H\delta\ 0.41030}$	0.134	0.0969	0.0719
$j_{75\ 4.6538}/j_{H\epsilon\ 0.39713}$	0.134	0.101	0.0774
$j_{85\ 3.7406}/j_{H8\ 0.38902}$	0.130	0.101	0.0790
$j_{95\ 3.2970}/j_{H9\ 0.38365}$	0.127	0.100	0.0797
$j_{10\,5\ 3.0392}/j_{H10\ 0.37990}$	0.125	0.0997	0.0801
Humphreys ($n \to 6$) line intensities relative to corresponding Balmer lines			
$j_{76\ 12.372}/j_{H\epsilon\ 0.39713}$	0.0855	0.0601	0.0435
$j_{86\ 7.5026}/j_{H8\ 0.38902}$	0.0867	0.0632	0.0471
$j_{96\ 5.9083}/j_{H9\ 0.38365}$	0.0850	0.0634	0.0481
$j_{10\,6\ 5.1287}/j_{H10\ 0.37990}$	0.0833	0.0632	0.0486

[a] Emissivities from Hummer & Storey (1987)

HI Continuum Spectrum