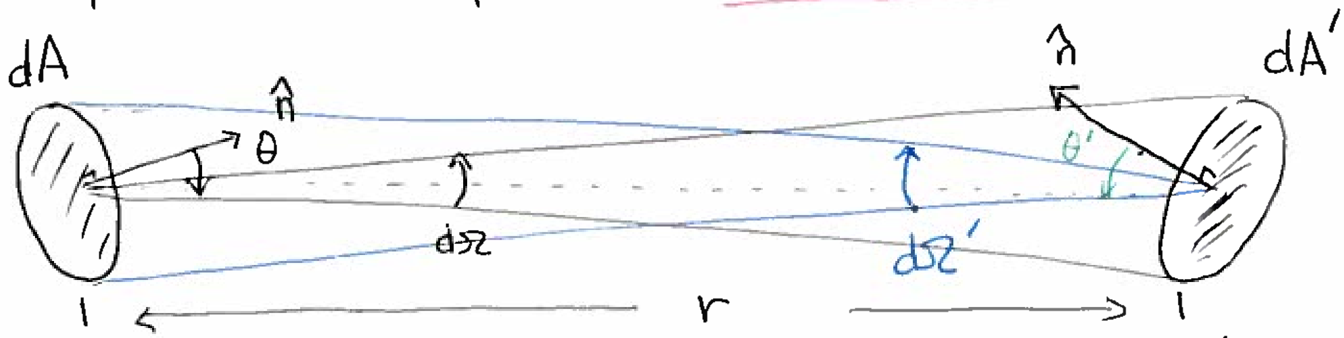
Photometric Concepts

Specific Intensity is Distance Independent



$$dE = I \cos\theta \, dA \, d\Omega \, dt \quad \text{with} \quad d\Omega = \frac{dA' \cos\theta'}{r^2}$$

If no energy lost between dA and dA'

$$dE = I' \cos\theta' \, dA' \, d\Omega' \, dt \quad \text{with} \quad d\Omega' = \frac{dA \cos\theta}{r^2}$$

$$I \cos\theta \, dA \cdot \frac{dA' \cos\theta'}{r^2} \, dt = I' \cos\theta' \, dA' \, \frac{dA \cos\theta}{r^2} \, dt$$

$$\Rightarrow \quad I = I'$$

Specific intensity remains CONSTANT in empty space.

Flux Density has inverse square dependence

$$F \sim \langle I \rangle \cdot \Omega \quad \text{and} \quad \Omega \sim \frac{1}{r^2}$$

$$\Rightarrow \quad F \sim \frac{1}{r^2}$$

Flux Density OR Flux = Rate of energy flowing through dA from all solid angles

$$F_\nu = \int_\Omega I_\nu \cdot \cos\theta \, d\Omega \qquad erg \cdot s^{-1} \cdot cm^{-2} \cdot Hz^{-1}$$

(Sometimes also written S_ν)

Note radio astronomers call F_ν Flux Density where the "density" refers to per Hz.

Radio unit: 1 Jansky = 1 Jy = 10^{-23} $erg \cdot s^{-1} \cdot cm^{-2} \cdot Hz^{-1}$

The flux is integrated over frequency (or wavelength) and has units of $erg \cdot s^{-1} \cdot cm^{-2}$

Flux $$F = \int_\nu \int_\Omega I_\nu \cos\theta \, d\Omega \, d\nu \qquad erg \cdot s^{-1} \cdot cm^{-2}$$

Power $$P = \int_{area} F \, dA \qquad erg \cdot s^{-1}$$

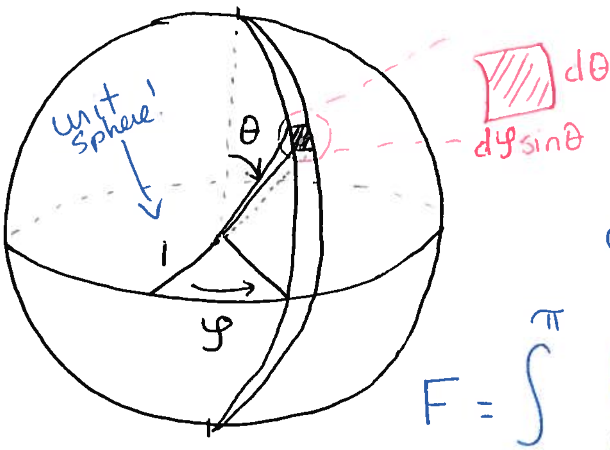
NOTE: 1 Watt = 10^{7} $erg \cdot s^{-1}$

Quantity	Definition	UNITS
Monochromatic Specific Intensity	$I_\nu \equiv \dfrac{dE}{dt \, dA\cos\theta \, d\Omega \, d\nu}$	$erg \cdot s^{-1} \cdot cm^{-2} \cdot ster^{-1} \cdot Hz^{-1}$
Total Specific Intensity	$I \equiv \int_0^\infty I_\nu \, d\nu$	$erg \cdot s^{-1} \cdot cm^{-2} \cdot ster^{-1}$
Flux Density	$F_\nu \equiv \int_\Omega I_\nu \cos\theta \, d\Omega$	$erg \cdot s^{-1} \cdot cm^{-2} \cdot Hz^{-1}$
Flux	$F \equiv \int_0^\infty F_\nu \, d\nu$	$erg \cdot s^{-1} \cdot cm^{-2}$
Power	$P \equiv \int_{area} F \, dA$	$erg \cdot s^{-1}$
Luminosity	$L \equiv \int_{\text{total surface area}} F^+ \, dA$ $\leftarrow$ emergent flux	$erg \cdot s^{-1}$

Let's calculate the total flux for an isotropic radiation field

$$I(\hat{e}_1) = \text{CONST} = I \qquad erg \cdot s^{-1} \cdot cm^{-2} \cdot ster^{-1}$$

$$\text{Flux} = F = \int_{\Omega} I \cos\theta \, d\Omega \qquad erg \cdot s^{-1} \cdot cm^{-2}$$



What is $d\Omega$? In spherical coordinates:

$$dA = \sin\theta \, d\varphi \, d\theta$$

$$d\Omega = \frac{dA}{r^2} = \frac{dA}{1} = \sin\theta \, d\theta \, d\varphi$$

$$F = \int_{\theta=0}^{\pi} \int_{\varphi=0}^{2\pi} I \cos\theta \sin\theta \, d\theta \, d\varphi$$

$$F = I \cdot \varphi \Big|_0^{2\pi} \cdot (-1) \int_{\theta=0}^{\pi} \cos\theta \, (-\sin\theta \, d\theta)$$

$$= 2\pi I \cdot \int_{\theta=\pi}^{0} \cos\theta \, d(\cos\theta)$$

$$= \cancel{2}\pi I \cdot \frac{1}{\cancel{2}} \cos^2\theta \Big|_{\pi}^{0} = \pi I \left(\cancelto{1}{\cos^2 0} - \cancelto{1}{\cos^2 \pi} \right) \quad 1 - 1 = 0$$

$\Rightarrow F = 0$!! There is zero net flux of radiation through dA $\Rightarrow$ equal amounts flow from all directions.

$\Rightarrow$ WE MUST CONSIDER DIRECTION!

EMERGENT Flux (i.e. the flux emerging from the surface of a star)

$$F^{+} = \int_{\theta=0}^{\pi/2} \int_{\varphi=0}^{2\pi} I \cos\theta \sin\theta \, d\theta \, d\varphi$$

$$1 - 0 = 1$$

$$= \pi I \cdot \cos^2\theta \Big|_{\pi/2}^{0} = \pi I \left(\cos^2 0 - \cos^2 \frac{\pi}{2} \right)$$

$$\boxed{F^{+} = \pi I}$$

Also called the "Astrophysical flux"

For a star, the luminosity is just the total integrated emergent flux from the star

$$L = \int F_*^{+} \, dA = 4\pi R_*^2 F_*^{+} \quad erg \cdot s^{-1}$$

surface area of star

Note, sometimes it is more convenient to work with flux densities (Hz^{-1}). So we also define the "Specific luminosity" as

$$L_\nu = \int F_\nu^{+} \, dA \quad erg \cdot s^{-1} \cdot Hz^{-1}$$

surface area