Clebsch-Gordon Series

Consider 2 angular momenta $\vec{l}_1$ and $\vec{l}_2$

The quantum numbers m_{l_1} and m_{l_2} vary from:

$-l_1 \le m_{l_1} \le +l_1$ $\qquad$ $-l_2 \le m_{l_2} \le +l_2$

Now $\vec{L} = \vec{l}_1 + \vec{l}_2$ and $M_L = m_{l_1} + m_{l_2}$

$-l_1 \le M_L - m_{l_2} \le +l_1$ $\qquad$ $-l_2 \le M_L - m_{l_1} \le +l_2$

Let M_L, m_{l_1}, and m_{l_2} assume their maximal values

MAX: $M_L = +L$ $\qquad$ $m_{l_1} = +l_1$ $\qquad$ $m_{l_2} = +l_2$

then we have:

$-l_1 \le L - l_2 \le +l_1$ $\qquad$ $-l_2 \le L - l_1 \le +l_2$

ADDING $+l_2$: $\qquad$ ADDING $+l_1$:

$l_2 - l_1 \le L \le l_1 + l_2$ $\qquad$ $l_1 - l_2 \le L \le l_1 + l_2$

SINCE $L \ge l_2 - l_1$ and $L \ge l_1 - l_2 \Rightarrow L \ge |l_1 - l_2|$

Therefore

$$|l_1 - l_2| \le L \le l_1 + l_2$$

SINCE L is an angular momentum, it can only change by integral values

$$L = l_1 + l_2,\ l_1 + l_2 - 1,\ \ldots,\ |l_1 - l_2|$$

Q.E.D.