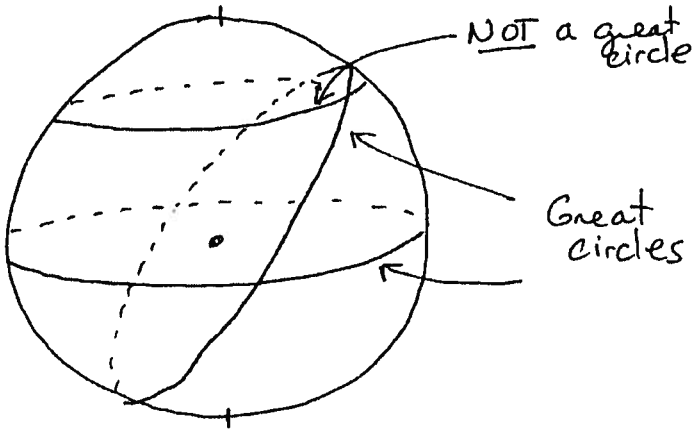
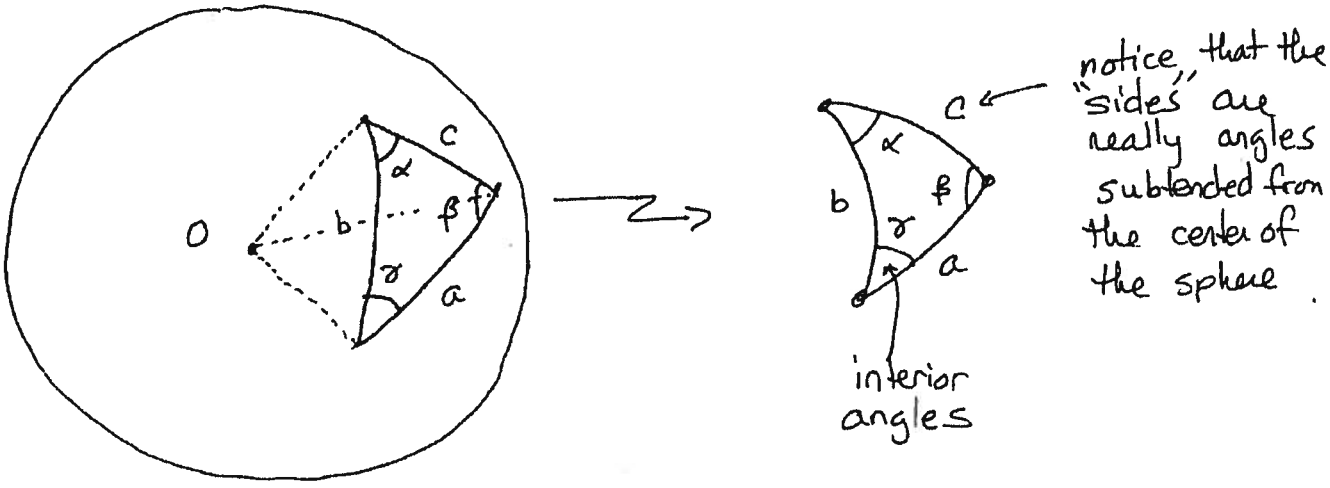
Spherical Trigonometry

Defn.: Great Circle ≡ a circle on the surface of a sphere whose center is the center of the sphere

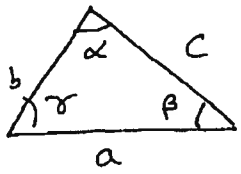


Defn: Spherical Triangle ≡ a triangle on the surface of a sphere whose sides are made of 3 great circles



Euclidean Geometry (Flat Space)

Law of Cosines



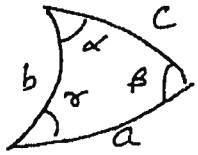
$$a^2 = b^2 + c^2 - 2ab\cos\alpha$$

Law of Sines

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

Spherical Geometry ("Non-Euclidean")

Law of Cosines



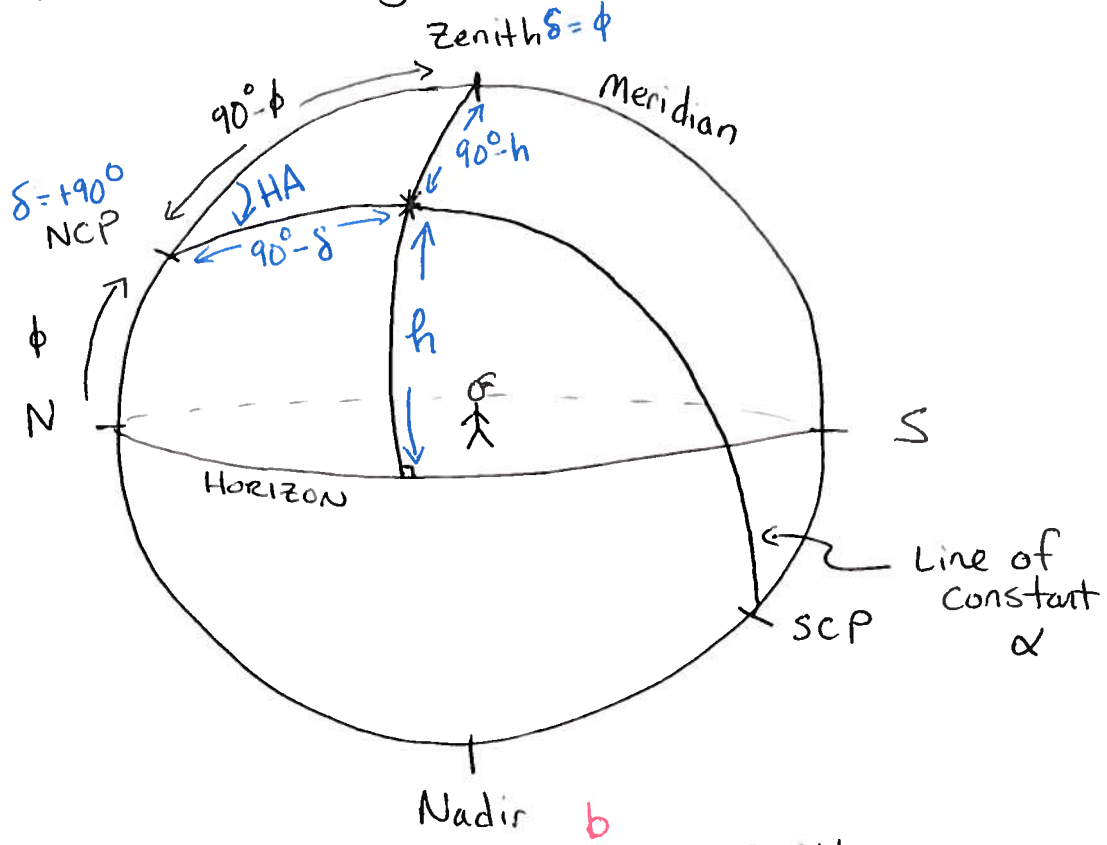
$$\cos a = \cos b \cos c + \sin b \sin c \cos\alpha$$

Law of Sines

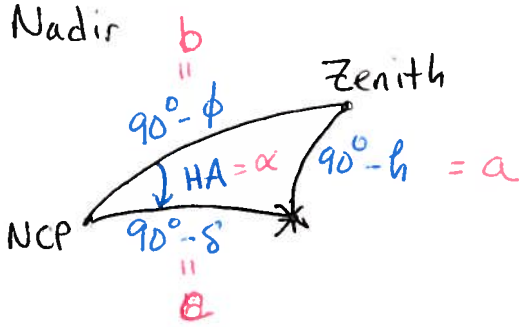
$$\frac{\sin\alpha}{\sin a} = \frac{\sin\beta}{\sin b} = \frac{\sin\gamma}{\sin c}$$

see online notes for a derivation of these equations or Fundamental Astronomy §2.1

Spherical Triangle Alt Az - RA Dec



Extracting our Triangle:



Law of Cosines: $\cos a = \cos b \cdot \cos c + \sin b \sin c \cos \alpha$

$\cos(90°-h) = \cos(90-\phi)\cos(90-\delta) + \sin(90-\phi)\sin(90-\delta)\cos HA$

USING TRIG IDENTITIES:

$$\sin h = \sin\phi \sin\delta + \cos\phi \cos\delta \cos HA$$

Remember $HA = LST - \alpha$

This equation allows you to calculate the elevation of a source with (α, δ) at any LST. Telescope control systems w/ Alt-Az mounts do this calc many times per second.