

Redshift

ASTR
250

λ_{obs} = observed wavelength

λ_{em} = emitted wavelength in rest frame of the object

$$z = \text{redshift} = \frac{\Delta\lambda}{\lambda_{em}} = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}}$$

NOTE: $z = \frac{\lambda_{obs}}{\lambda_{em}} - 1$

$$1 + z = \frac{\lambda_{obs}}{\lambda_{em}}$$

In general, there are 3 main sources to redshifts:

① Doppler shift due to "peculiar velocities"

$$1 + z = \frac{\lambda_{obs}}{\lambda_{em}} = \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}}$$

NOTE: in relativity " $\frac{v}{c} \equiv \beta$ "

Special relativity for motion purely along line of sight

Note for $v \ll c$ we can Taylor expand

$$\left(1 + \frac{v}{c}\right)^{1/2} \sim 1 + \frac{1}{2} \frac{v}{c} + O\left(\frac{v^2}{c^2}\right) \dots$$

$$\left(1 - \frac{v}{c}\right)^{1/2} \sim 1 - \frac{1}{2} \frac{v}{c} + O\left(\frac{v^2}{c^2}\right) \dots$$

$$1 + z = \left(1 + \frac{v}{c}\right)^{1/2} \left(1 - \frac{v}{c}\right)^{-1/2} \sim \left(1 + \frac{1}{2} \frac{v}{c}\right) \left(1 + \frac{1}{2} \frac{v}{c}\right)$$

$$\sim 1 + \frac{v}{c} \Rightarrow v \sim c \cdot z$$

(2) Gravitational redshift - Can be observed in strong gravitational field.

$$\frac{\nu_{\text{obs}}}{\nu_{\text{em}}} = \sqrt{1 - \frac{2GM}{c^2 R}}$$

General relativity for a non-rotating metric

(3) Cosmological redshift due to expansion of spacetime.

Hubble law $v = H_0 \cdot D$

and $1 + z = \frac{a_{\text{now}}}{a(t)}$ ← "scale factor" that is a fundamental measure of length in spacetime.

we will talk about this in detail next class...